M2S1 - ASSESSED COURSEWORK 1: Hints for part (c)

For continuous random variable Z with pdf f_Z and cdf F_Z, from first principles, we have

$$E_{f_Z}[g(Z)] = \int g(z) f_Z(z) dz$$

and

$$F_Z(z) = P[Z \leq z] = \int_{-\infty}^{z} f_Z(s) ds \qquad \Longleftrightarrow \qquad f_Z(z) = \frac{d}{ds} \{F_Z(s)\}_{s=z}$$

so putting these together we get

$$E_{f_Z}[g(Z)] = \int g(z) \frac{d}{ds} \{F_Z(s)\}_{s=z} dz.$$

Secondly, it is (usually) legitimate to exchange the order of **differentiation** and summation, that is, if

$$H(z) = \sum_k H_k(z)$$

then

$$h(z) = \frac{d}{ds} \{H(s)\}_{s=z} \quad = \quad \frac{d}{ds} \left\{ \sum_k H_k(s) \right\}_{s=z} = \sum_k \frac{d}{ds} \{H_k(s)\}_{s=z} = \sum_k h_k(z)$$

say. Similarly, it is (usually) legitimate to exchange the order of **integration** and summation.

Finally, if $X_1, \ldots, X_n$ are independent and identically distributed random variables, and

$$Y = \sum_{i=1}^{n} X_i$$

then lecture notes give results for both the **mgf** and **expectation** of Y in terms of the mgf and expectation of $X_1, \ldots, X_n$.

Using these results, and the results given in lectures, you should find that you can compute the required expectation and variance of variable T using partition given in the original hint, after computing **only**

- $E_{f_X}[X]$,
- $E_{f_X}[X^2]$, and/or
- M_X.

where X is the variable from part (b). **You should find that you do not have to compute the pdf or cdf of T.**