M1S TUTORIAL SHEET : WEEK 8

PROPERTIES OF CONTINUOUS PROBABILITY DISTRIBUTIONS

The specification of a continuous probability distribution continuous random variable X is of X is achieved via the cumulative distribution function (cdf) F_X, or via the probability density function (pdf), f_X, where

$$F_X(x) \equiv \mathrm{P}[X \leq x] \qquad f_X(x) = \frac{d}{dt}\{F_X(t)\}_{t=x}$$

for real-values of x. Thus the fundamental relationship between F_X and f_X is given by

$$F_X(x) = \int_{-\infty}^{x} f_X(t)\, dt$$

This is analogous to the relationship between cumulative distribution function and probability mass function in the discrete case

The probability axioms dictate the properties of F_X, namely that

(i) F_X is non-decreasing (ii) F_X is continuous

(iii) $\lim_{x \to -\infty} F_X(x) = 0$ (iv) $\lim_{x \to \infty} F_X(x) = 1$

which, in turn, implies that the probability density function f_X must satisfy

(i) $f_X(x) \geq 0, \ \forall x$ (ii) $\int_{-\infty}^{\infty} f_X(x)\, dx = 1$

Verify these properties for the following cumulative distribution functions, and find the corresponding density functions.

(a) $F_X(x) = \dfrac{e^x}{1+e^x}, \qquad x \in \mathbb{R}$

(b) $F_X(x) = \exp\left\{-e^{-\lambda x^{\alpha}}\right\}, \qquad$ for $x \in \mathbb{R}, \ \alpha, \lambda > 0$

(c) $F_X(x) = 1 - \exp\{-\lambda x^{\alpha}\}, \qquad$ for $x \in \mathbb{R}^+$, and zero otherwise, $\alpha, \lambda > 0$

Now consider the density function f_X specified for random variable X taking values on range $\mathbb{X} = [0, 1)$;

$$f_X(x) = \begin{cases} 2x & 0 \leq x \leq 1/2 \\ 6(1-x) & 1/2 < x < 1 \end{cases}$$

and zero otherwise. Sketch this density function, and find and sketch the corresponding distribution function.