M1S TUTORIAL SHEET : WEEK 5

In many situations, a probability calculation can be reduced to an exercise in counting equally likely sample outcomes using combinatorial techniques. If the sample space comprises n_Ω equally likely outcomes, and event E represents a collection of n_E of them, then we can legitimately define $\mathrm{P}(E)$ by

$$\mathrm{P}(E) = \frac{n_E}{n_\Omega},$$

and so the probability calculation only requires enumeration of n_E and n_Ω.

1. The Hypergeometric Probability formulae as given in lectures are two alternative ways of calculating a probability in a specific classical probability sampling problem. Verify by writing out the factorial expressions of

$$\frac{\binom{n}{r}\binom{N-n}{R-r}}{\binom{N}{R}} \quad \text{and} \quad \frac{\binom{R}{r}\binom{N-R}{n-r}}{\binom{N}{n}}$$

that there is equality in the two formulae. The two justifications are obtained as follows:

Method A: Consider choosing items in the population to label Type I and Type II, broken down by whether they are IN the sample or OUT of the sample. The denominator is the total number of ways of choosing R from N items to label Type I. The numerator is the number of ways of choosing r from n items IN the sample to be Type I items, multiplied by the number of ways of choosing the $R-r$ from $N-R$ items OUT of the sample to be Type II items.

Method B: Consider items to be IN the sample, broken down by Type. The denominator is the total number of ways of choosing n items from N to be IN the sample. The numerator is the number of ways of choosing r Type I items from R to be IN the sample, multiplied by the number of ways of choosing the $n-r$ Type II items from $N-R$ to be IN the sample.

2. Use counting approaches in the solution of the following problems;

(a) *The Birthday Problem*: What is the probability that, in a class of N students, no two students have the same birthday, assuming that birthdays in the class are evenly distributed through the year ? Evaluate this probability for a few values of N.

(b) Six fair dice are rolled. What is the probability that a full set of scores $\{1, 2, 3, 4, 5, 6\}$ is obtained ?

(c) Hands in poker are ranked (from lowest to highest) as follows:

HAND	DESCRIPTION
One Pair	Two cards of the same denomination
Two Pairs	Two pairs of two cards of the same denomination
Three of a kind	Three cards of the same denomination
Straight	Five cards having consecutive denominations
Flush	Five cards having the same suit
Full house	Three cards of one denomination and two of another
Four of a kind	Four cards of the same denomination
Straight flush	Five cards having the same suit and consecutive denominations
Royal flush	Ace, King, Queen, Jack, 10 in the same suit

Show that there is a valid probabilistic reason for such a ranking.

Note: There is a conditional probability solution, involving the general multiplication rule for events, for each of these problems; you may find it easier to use that approach.