

M1S : EXERCISE SHEET 1 : SOLUTIONS

1. (a) $A \cap B' \cap C'$
 (b) $A \cap B \cap C'$
 (c) $A \cap B \cap C$
 (d) $A \cup B \cup C$
 (e) $(A \cap B \cap C') \cup (A \cap B' \cap C) \cup (A' \cap B \cap C) \cup (A \cap B \cap C)$
 (f) $(A \cap B' \cap C') \cup (A' \cap B \cap C') \cup (A' \cap B' \cap C)$
 (g) $(A \cap B \cap C') \cup (A \cap B' \cap C) \cup (A' \cap B \cap C)$
 (h) $A' \cap B' \cap C'$
 (i) $(A \cap B \cap C)' \equiv A' \cup B' \cup C'$

2. (a) TRUE : distributivity
 (b) TRUE : $(A \cap B') \cup B = (B \cup A) \cap (B \cup B') = (B \cup A) \cap \Omega$.
 (c) TRUE : distributivity
 (d) FALSE : $\omega \in LHS \implies \omega \in C$, but $\omega \in RHS \implies \omega \in C'$ (CONTRADICTION).
 (e) TRUE : $\omega \in LHS \implies \omega \in B$ and $\omega \in B' \implies$ no such ω exists.

3. (a) B'
 (b) $A' \cap B$
 (c) $(A \cap C) \cup (A' \cap B)$

4. (a) $\Omega = \{FF, SFF, SSFF, FSFF, SSSS, SSSF, SSFS, SFSS, FSSS, FSFS, SFSS, FSSF\}$
 (b) $\{SSSS, SSFS, SFSS, FSSS, FSFS\}$

5. (a) $\Omega = \{A_i : i = 1, \dots, n - r + 1\}$, where

$$A_i \equiv \text{“find first defective on } i\text{th inspection”}$$

 (b) $\Omega = \{B_i : i = r, r + 1, \dots, n\}$, where

$$B_i \equiv \text{“find last defective on } i\text{th inspection”}$$

6. (a) $[A_1 \cup (A'_1 \cap A_2)] \cap B \cap C \cap D \cap E$
 (b) Replace B in (a) by the DISJOINT UNION

$$(B_1 \cap B_2 \cap B'_3) \cup (B_1 \cap B'_2 \cap B_3) \cup (B'_1 \cap B_2 \cap B_3) \cup (B_1 \cap B_2 \cap B_3)$$

NOTE: Can use Venn Diagrams as an aid to solving these problems

TUTORIAL SHEET WEEK 2 : SOLUTIONS

1. (a) $\Omega = \{FF, FM, MF, MM\}$

(b) $A = \{FF\}, B = \{MM\}, C = \{FM, MF, MM\}$

$$\begin{aligned} \Rightarrow \quad & A \cap B = \emptyset \\ & A \cup B = \{FF, MM\} \\ & A \cap C = \emptyset \\ & A \cup C = \Omega \\ & B \cap C = \{MM\} \\ & B \cup C = C \\ & B' \cup C = \{FM, MF\} \end{aligned}$$

2. Events are found by inspection of sample outcomes in Ω (form Ω as a two-way, 6×6 table whose entries are ordered pairs of scores). Can list elements in each case :

$$\begin{aligned} A &= \{(1, 2), (2, 2), (3, 2), (4, 2), (5, 2), (6, 2), \\ &\quad (1, 4), (2, 4), (3, 4), (4, 4), (5, 4), (6, 4), \\ &\quad (1, 6), (2, 6), (3, 6), (4, 6), (5, 6), (6, 6)\} \end{aligned}$$

$$C' = \{(2, 2), (2, 4), (2, 6), (4, 2), (4, 4), (4, 6), (6, 2), (6, 4), (6, 6)\}$$

$$A \cap B = \{(2, 2), (4, 2), (6, 2), (2, 4), (4, 4), (6, 4), (2, 6), (4, 6), (6, 6)\}$$

$$A \cap B' = \{(1, 2), (3, 2), (5, 2), (1, 4), (3, 4), (5, 4), (1, 6), (3, 6), (5, 6)\}$$

$$A' \cup B = \Omega \setminus \{(1, 2), (1, 4), (1, 6), (3, 2), (3, 4), (3, 6), (5, 2), (5, 4), (5, 6)\}$$

$$\begin{aligned} A' \cap C &= \{(1, 1), (2, 1), (3, 1), (4, 1), (5, 1), (6, 1), \\ &\quad (1, 3), (2, 3), (3, 3), (4, 3), (5, 3), (6, 3), \\ &\quad (1, 5), (2, 5), (3, 5), (4, 5), (5, 5), (6, 5)\} \end{aligned}$$

and the required events have 18, 9, 9, 9, 27 and 18 elements respectively.

3. Label defectives D_1, D_2 , and others G_1, G_2, G_3 . Then Ω is the set

$$\{(D_1, D_2), (D_1, G_1), (D_1, G_2), (D_1, G_3), (D_2, G_1), (D_2, G_2), (D_2, G_3), (G_1, G_2), (G_1, G_3), (G_2, G_3)\}$$

and

$$A = \{(G_1, G_2), (G_1, G_3), (G_2, G_3)\}$$