M1S PROBABILITY AND STATISTICS I
2001 SYLLABUS

CHAPTER 1 : Sample Spaces and Events

1.1 Representing uncertainty in experimental contexts

1.2 Manipulating Collections of Sample Outcomes
- Sample outcomes
- Sample spaces (discrete/continuous, finite/countable/uncountable
- Events - occurrence, the Certain Event, the Impossible Event
- Set theory notation

1.3 Operations of Set Theory
- Binary Operations - Complement/Union/Intersection
- Exhaustive/Exclusive Events
- Elementary results - De Morgan's Laws
- Extensions of union/intersection ideas to more than two events - finite/countable unions
- Associative/Distributive Laws
- Representations of Complex Systems - component networks
- Disjoint unions/Partitions
- Construction of Partitions

CHAPTER 2 : Probability : Definitions, Interpretations, Basic Laws.

2.1 The Meaning of Probability
- Interpretations of the Probability Function (relative frequency,classical,subjective)
- Simple examples (0/1 experiments e.g. coin/drawing pin)

2.2 The Mathematical Rules of Probability: The Probability Axioms
- The Probability Axioms
- Extensions to Axiom III to finite and countable collections of events

2.3 Corollaries to the Axioms
- Probabilities for complement events
- Probability of a union for non-exclusive events (The General Addition Rule)

CHAPTER 3 : Conditional Probability: Conditioning on New Information.

- Conditional Probability - definition, interpretation
- Proof that the conditional probability operator obeys the probability axioms
- Justification in terms of different probability interpretations
- Examples of simple calculations
- Independence for two events/ Mutual independence for more than two events
- The General Multiplication (or Chain) Rule for Events (link to probability trees)

3.1 The Theorem of Total Probability
- Statement and Proof of Theorem: Interpretation and Key elements (i.e. partition, decomposition of probs.)
- Application in simple and more complicated examples

3.2 Bayes Theorem
- Statement and Proof of Theorem for two events/more than two events
- Interpretation : Distinction between $P(A|B)$ and $P(B|A)$
- Examples e.g. Medical Diagnosis, Simpson's Paradox
- Prior and Posterior Odds
- Conditional probability for more than two events; Conditional Independence

CHAPTER 4 : Counting Techniques: Combinatorics.

Enumeration for Equally Likely Outcomes : Combinatorics Problems (Sampling from a Finite Population, Occupancy Problems, Urn Models)

4.1 Counting Operations : Basic Methods and Terminology
Multiplication principle, Factorials
Distinguishable/Indistinguishable Objects;
Sampling with/without replacement;
Ordered/unordered outcomes.
Permutations/Combinations
Binary sequence representations

4.2 Combinatorial Identities

4.3 The Hypergeometric Formula
The Hypergeometric Formula : context and two alternative justifications
Combinatorial/conditional probability justification
Uses for the Hypergeometric formula in probability calculations (for N, R, n or r varying)
Examples e.g. Lottery, Capture-Recapture, Survey Sampling

4.4 Combinatorial Partitioning : The Partition formula
The use of partitioning in a combinatorial context
Examples of Partitions (e.g. Poker hands)

4.5 Occupancy Problems (Distribution Problems)
Counting Techniques for Occupancy Problems involving
- Distinguishable objects - as a partitioning problem.
- Indistinguishable objects - using a binary sequence representation
Examples e.g. The Birthday Problem

CHAPTER 5 : Discrete Random Variables and Distributions

5.1 Random Variables
General definition
Discrete case (countable range)

5.2 Probability Mass Function
Definition, Notation, Interpretation Properties
Expectation.
Probability Generating Function (PGF) and its use.

5.3 Discrete Cumulative Distribution Function
Definition, Notation, Interpretation
Properties
Connection with the probability mass function (summation/differencing)

5.4 Bernoulli Distribution

5.5 Binomial Distribution
Definition, experimental context, interpretation
Mass function Limiting behaviour as $n \to \infty, \theta \to 0$ with $n\theta = \lambda$ constant

5.6 Poisson Distribution
Definition, experimental context, interpretation
Mass function
Connection to the Poisson Process

5.7 Geometric Distribution
Mass function and CDF

5.8 Negative Binomial Distribution
Definition, experimental context, interpretation
Mass function
Connection to Geometric distribution

5.9 Hypergeometric Distribution
Definition, experimental context, interpretation
Mass function
Connection to Binomial distribution

5.10 Probability Generating Functions
Definition
Uses: calculations for sums of independent random variables.
How to use PGF to compute a pmf.
Calculation of PGFs of standard distributions

CHAPTER 6 : Continuous Random Variables and Distributions

6.1 Continuous Random Variables
General definition
Continuous probability specifications; context
6.2 Continuous Cumulative Distribution Function
Definition, Notation, Interpretation
Properties

6.3 Probability Density Function
Definition, Notation, Interpretation
Properties; Expectation and Variance.
Connection with the continuous CDF function (integration/differentiation)
A Generating Function for continuous variables and its uses.

6.4 Continuous Uniform Distribution

6.5 Exponential Distribution
Definition, experimental context, interpretation
PDF and CDF
Connection to Poisson and Poisson Process

6.6 Gamma Distribution
Definition, interpretation
The GAMMA function and properties
PDF
Special case: Chi-squared distribution ($\alpha = n/2, \beta = 1/2$) χ^2_n
Connection to Exponential

6.7 Normal Distribution
Definition, experimental context, interpretation
PDF in standard and non-standard cases
Linear (location/scale) transformations
Connection to Gamma

CHAPTER 7 : Transformations

Transformations of random variables ($Y = g(X)$)
Location/scale transforms
General transformation technique
Transformation Theorem for g 1-1

CHAPTER 8 : Generalized Expectation

Further study of expectations of a discrete/continuous random variable
Expectations for general functions of random variables
Properties and special cases
Linearity of Expectations
Probability and Moment Generating Functions as Expectations
Link between PGF and MGF
Calculations of PGF/MGF for standard distributions
Key results and uses of PGFs/MGFs
The Central Limit Theorem

CHAPTER 9 : Joint Distributions

General introduction to joint discrete/continuous distributions
Independence for random variables
Expectations for Sums of Random Variables