M1S PROBABILITY AND STATISTICS I

SYLLABUS

CHAPTER 1 : Sample Spaces and Events

1.1 Representing uncertainty in experimental contexts

1.2 Manipulating Collections of Sample Outcomes
Sample outcomes
Sample spaces (discrete/continuous, finite/countable/uncountable
Events - occurrence, the Certain Event, the Impossible Event
Set theory notation

1.3 Operations of Set Theory
Binary Operations - Complement/Union/Intersection
Exhaustive/Exclusive Events
Elementary results - De Morgan's Laws
Extensions of union/intersection ideas to more than two events - finite/countable unions
Associative/Distributive Laws
Representations of Complex Systems - component networks
Disjoint unions/Partitions
Construction of Partitions

CHAPTER 2 : Probability : Definitions, Interpretations, Basic Laws.

2.1 The Meaning of Probability
Interpretations of the Probability Function (relative frequency,classical,subjective)
Simple examples (0/1 experiments e.g. coin/drawing pin)
Specification of probabilities via Odds

2.2 The Mathematical Rules of Probability: The Probability Axioms
The Probability Axioms
Extensions to Axiom III to finite and countable collections of events

2.3 Corollaries to the Axioms
Probabilities for complement events
Probability of a union for non-exclusive events (The General Addition Rule)

CHAPTER 3 : Conditional Probability: Conditioning on New Information.

Conditional Probability - definition, interpretation
Examples of simple calculations
Independence for two events/ Mutual independence for more than two events
The General Multiplication (or Chain) Rule for Events (link to probability trees)

3.1 The Theorem of Total Probability
Statement and Proof of Theorem
Interpretation and Key elements
Application in simple and more complicated examples

3.2 Bayes Theorem
Statement and Proof of Theorem for two events/more than two events
Interpretation : Distinction between $P(A|B)$ and $P(B|A)$
Examples e.g. Medical Diagnosis, Simpson's Paradox
Prior and Posterior Odds
Conditional probability for more than two events
Conditional Independence

CHAPTER 4 : Counting Techniques: Combinatorics.

Enumeration for Equally Likely Outcomes : Combinatorics Problems (Sampling from a Finite Population, Occupancy Problems, Urn Models)

4.1 Counting Operations : Basic Methods and Terminology
Multiplication principle, Factorials
Distinguishable/Indistinguishable Objects;
Sampling with/without replacement;
Ordered/unordered outcomes.
Permutations/Combinations
Binary sequence representations

4.2 Combinatorial Identities

4.3 Partitioning : The Partition formula
Examples of Partitions (e.g. Poker hands)

4.4 Occupancy Problems (Distribution Problems)
Counting Techniques for Occupancy Problems involving
- Distinguishable objects - as a partitioning problem.
- Indistinguishable objects - using a binary sequence representation
Examples e.g. The Birthday Problem

4.5 Urn Models
The Hypergeometric Formula : two alternative justifications
Combinatorial/conditional probability justification
Uses for the Hypergeometric formula in probability calculations (for N, R, n or r varying)
Examples e.g. Fisher's Exact Test

4.6 Generating Functions for Combinatorics Calculations
Generating functions: definitions and uses
Solution of combinatorics problems using generating functions

CHAPTER 5 : Discrete Random Variables and Distributions

5.1 Random Variables
General definition
Discrete case (countable range)

5.2 Probability Mass Function
Definition, Notation, Interpretation
Properties; General Expectation and Variance; Examples.

5.3 Discrete Cumulative Distribution Function
Definition, Notation, Interpretation
Properties
Connection with the probability mass function (summation/differencing)

5.4 Bernoulli Distribution

5.5 Binomial Distribution
Definition, experimental context, interpretation
Mass function Limiting behaviour as $n \to \infty, \theta \to 0$ with $n\theta = \lambda$ constant

5.6 Poisson Distribution
Definition, experimental context, interpretation
Mass function
Connection to the Poisson Process

5.7 Geometric Distribution
Mass function and CDF

5.8 Negative Binomial Distribution
Definition, experimental context, interpretation
Mass function
Connection to Geometric distribution

5.9 Hypergeometric Distribution
Definition, experimental context, interpretation
Mass function
Connection to Binomial distribution

5.10 Probability Generating Functions
Definition
Uses: calculations for sums of independent random variables.
Formulae for PGFs of standard distributions (eg. Geometric, Binomial, Poisson)

CHAPTER 6 : Continuous Random Variables and Distributions

6.1 Continuous Random Variables
General definition
Continuous probability specifications as the limit of discrete specifications

6.2 Continuous Cumulative Distribution Function
Definition, Notation, Interpretation
Properties

6.3 Probability Density Function
Definition, Notation, Interpretation
Properties; Expectation and Variance.
Connection with the continuous CDF function (integration/differentiation)

6.4 Continuous Uniform Distribution

6.5 Exponential Distribution
Definition, experimental context, interpretation
PDF and CDF
Connection to Poisson and Poisson Process
Connection to Uniform

6.6 Gamma Distribution
Definition, interpretation
The GAMMA function and properties
PDF
Special case: Chi-squared distribution ($\alpha = n/2, \beta = 1/2$) χ^2_n
Connection to Exponential
Connection to Normal

6.7 Normal Distribution
Definition, experimental context, interpretation
PDF in standard and non–standard cases
Linear transformations

CHAPTER 7 : Transformations

Transformations of random variables ($Y = g(X)$)
General transformation technique
Transformation Theorem for g 1-1

CHAPTER 8 : Expectation

Further study of expectations of a discrete/continuous random variable
Expectations for general functions of random variables
Properties of Expectations
Linearity of Expectations
Probability and Moment Generating Functions as Expectations
Link between PGF and MGF
Calculations of PGF/MGF for standard distributions
Key results and uses of PGFs/MGFs

CHAPTER 9 : Joint Distributions

General joint discrete/continuous distributions
Discrete joint/marginal/conditional mass functions
Continuous joint/marginal/conditional density functions
Independence for random variables
Conditional Expectations
Bivariate Expectations
The Law of Iterated Expectations
Covariance/Correlation
The Convolution Formula
Expectations (and other calculation techniques) for Sums of Random Variables
The Central Limit Theorem