M1S : KEY DEFINITIONS, RESULTS AND TECHNIQUES

CHAPTER 1: SAMPLE SPACES AND EVENTS

- sample space
- sample outcome
- event
- certain event/impossible event
- set operations member of/subset/complement/union/intersection
- mutually exclusive events, exhaustive events
- De Morgan's Laws
- associativity/distributivity
- **TECHNIQUE : representation of complex events via simpler ones** using set manipulations
- partitions: definitions, constructions and uses

CHAPTER 2: PROBABILITY: DEFINITIONS, INTERPRETATIONS & LAWS

- mathematical definition (set function)
- interpretations (FREQUENTIST/CLASSICAL/SUBJECTIVE)
- the Probability Axioms, extensions and corollaries
- **TECHNIQUE: proof from first principles of probability results** e.g the general addition rule
- the importance of mutually exclusive events and partitions in probability calculations

CHAPTER 3: CONDITIONAL PROBABILITY

- definition and interpretation of conditional probability
- proof that the conditional probability operator obeys the (ordinary) probability axioms
- medical screening example
- independence and mutual independence
- conditional independence
- the general multiplication rule
- the Theorem of Total Probability; statement and proof, interpretation, use
- partitions and the Theorem of Total Probability
- Bayes Theorem; statement and proof, interpretation, use
- Bayes Theorem in the classic medical screening /testing problem
- odds

CHAPTER 4: COMBINATORICS AND PROBABILITY

- use in classical probability context
- general counting procedures: the multiplication principle
- sampling from a finitie population
- with/without replacement; ordered/unordered samples; distinguishable/indistinguishable items
- permutations and combinations
- binary sequence representations
- combinatorial identities
- hypergeometric probability: context, interpretation, use
- uses of hypergeometric formula in Total Probability/Bayes Theorem questions
- partition formula; proof of form by sequential partitioning, elementary examples (e.g. dice, poker hands etc)
- Occupancy problems: distinguishable/indistinguishable items
- Occupancy Problems Examples: n items to n boxes; r items to n boxes (The Birthday Problem)

CHAPTER 5: DISCRETE RANDOM VARIABLES & PROBABILITY DISTRIBUTIONS

- random variables; interpretation, ranges
- discrete random variables
- the probability mass function (PMF); definition and properties; construction of pmfs
- the Probability Generating Function (PGF); definition and uses
- expectation; definition and interpretation
- the cumulative distribution function (CDF); definition, properties, uses
- **TECHNIQUE: Calculations for discrete distributions (PMF/CDF, PGF, EXPECTATION)**
- Bernoulli distribution
- Binomial distribution
- Poisson distribution (connection to - but not knowledge of - the Poisson process)
- Geometric distribution
- Negative Binomial distribution

(for each distribution, you should know the experimental context, how to perform standard calculations like pgf or expectation calculations, and the connections between the various distributions)

- probability generating functions: proofs of key results, and uses for expectation calculation, sums of independent random variables
- **TECHNIQUE: Use of pgfs**

CHAPTER 6: CONTINUOUS RANDOM VARIABLES

- continuous cumulative distribution function (continuous CDF); definition, interpretation, properties
- probability density function (PDF): definition, connection to CDF, constructing pdfs
- **TECHNIQUE : Calculations for continuous probability models (CDF $\leftrightarrow$ PDF, normalizing constants etc)**
- Expectation
- A Generating Function for continuous random variables and its uses
- Uniform distribution
- Exponential distribution
- Gamma distribution (and the Gamma Function) and the Chi-Squared distribution
- Normal distribution

(for each distribution, you should know the experimental context, how to perform standard calculations like pgf or expectation calculations, and the connections between the various distributions. You should also know about SCALE and LOCATION/SCALE transformations, and how they can be used to generalize standard distributions)

CHAPTER 7: TRANSFORMATIONS

- **TECHNIQUE: how to compute the distribution of a general function of a random variable, from first principles and/or using the general transformation result**
- Location-Scale and other 1-1 transformations
- Simple non 1-1 transformations (e.g. $Y = X^2, Y = X(1 - X)$ etc.)

CHAPTER 8: GENERALIZED EXPECTATIONS

- The definition of an expectation of a function of a random variable
- *The Law of the Unconscious Statistician*
- Properties/Special Cases
- Moments/Central Moments
- Variance
- Generating functions as geneneral expectations
- The Moment Generating Function (MGF) and its uses; connection of MGF to PGF
- Key MGF results and proofs
- Central Limit Theorem (context, not details)

CHAPTER 9: JOINT DISTRIBUTIONS

- Expectation of a sum (or linear combination) of (independent) random variables

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GENERAL KNOWLEDGE

- the binomial/negative binomial/geometric progression/exponential power series summations
- techniques of calculus

(you will be given the forms of pmfs/pdfs for the distributions we studied, but not any other quantities like pgfs, expectations, variances, so you should know how to calculate them),

THE FIVE MOST IMPORTANT ELEMENTS OF PROBABILITY THEORY (so far ...)

1. The Probability Axioms
2. Conditional Probability
3. Theorem of Total Probability and Bayes Theorem
4. Discrete and Continuous distributions
5. Expectation

SHORT QUIZ

1. Can a pair of events be both **mutually exclusive** and **independent** ?
2. What are the key components of the Theorem of Total Probability ?
3. What is Bayes Theorem used for, and how is it connected to the conditional probability definition ?
4. What is the probability that a single selection of six numbers matches the balls drawn in the National Lottery ?
5. Explain how permutations and combinations are used in classical probability calculations.
6. If presented with a probability distribution, how could you calculate the expectation of the distribution ?
7. A Poisson process model is assumed for events occurring through continuous time. List all the probability distributions that can be connected to this basic model.
8. What do you know about the Gamma distribution ?
9. How are the PGF and MGF of a continuous distribution related ?
10. If $X_1, ..., X_n$ are iid random variables, and $Y = \sum_{i=1}^{n} X_i$, how could you calculate

(i) the PMF of Y, if all variables are discrete
(ii) the expectation of Y
(iii) an approximation to the probability distribution of Y