M1S - Probability and Statistics I : Course Summary and Examination Hints

The course is intended as an introduction to mathematical components of probability theory, with reference to statistics. In my opinion, the key components of this theory are

- The set theory representation of experiments, outcomes, and events
- Partitions
- Probability and the Probability Axioms
- Conditional Probability
- The Theorem of Total Probability
- Bayes Theorem
- Combinations, Permutations, The Hypergeometric distribution, Combinatorial Partitions
- Discrete Random Variables, their distributions and properties (pmf, cdf, pgf, mgf, moments)
- Continuous Random Variables, their distributions and properties (pdf, cdf, mgf, moments)
- Calculation tricks (for expectation, moments etc)
- Distributions of special interest (Bernoulli, Binomial Poisson,..., Exponential, Gamma, Normal,...) and the connections between them
- Transformations of Random Variables
- Generalized Expectation
- Properties of PGFs and MGFs

In even more concise form, it is fairly accurate to say that everything in the course follows from a knowledge and understanding of the Probability Axioms, Conditional Probability, and the two Probability Theorems. The rest of the material is really just examples and special cases. There are some specific tricks and constructions that we met repeatedly, for example, sums of iid variables, results to do with generating functions, summation/integration tricks for expectations, like

$$f_X(x) = cf(x) \qquad \Longleftrightarrow \qquad \int f(x)\,dx = \frac{1}{c}$$

and how to sum/integrate functions that are proportional to pmfs/pdfs, and the "first principles approach" to calculation of the distribution of a transformation of a random variable, that starts

$$Y = g(X) \Rightarrow F_Y(y) = \mathrm{P}\left[Y \leq y\right] = \mathrm{P}\left[g(X) \leq y\right]$$

and so on. It is also useful to know about the experimental contexts of, and connections between, the special distributions that we studied in Chapters 5 and 6, and, if you can, remember the forms of their generating functions, moments etc (but you should be able to calculate them even if you cannot remember them).

Finally, some general hints. In the exam, if you are asked explictly to show, prove, or derive a particular result, you must do this in full detail. If you are asked to state a result, you only have to state it. If you use a result in any calculation or derivation that has been proved or stated in lectures, you may state and use the result without proof unless explictly told otherwise. More generally, before attempting a question have a look through to the end to see what the stream of ideas leading to; this may have the effect of reminding you of a specific question that you have studied.

Each of the five questions on the exam will contain a significant proportion of **bookwork** based on knowledge of the basic definitions, results and theorems contained in the course. The remainder of the question will usually require an **application of a fundamental technique** to a particular problem of a similar nature the material contained on the Exercise and Assessed worksheets. Usually the two parts of the question will be related. Markers are looking for evidence of (a) revision of material relevant to the question, and (b) understanding.of the material.

Overall, if you've revised the bookwork material, and managed to complete the all of the (extensive) Exercises and Assessed Coursework material (and can do so without reference to the solutions or Notes) then you should find the examination reasonably straightforward.

Some general points: do not write in red or green, and write as clearly as possible - rarely are marks deducted for poor presentation, but you can only lose marks through lack of clarity.

PLEASE ASK IF YOU NEED ANY FURTHER HELP OR CLARIFICATION

DA Stephens
d.stephens@ic.ac.uk