M1S : EXERCISES 5

DISCRETE PROBABILITY DISTRIBUTIONS

1. (a) The birthdays of 500 pupils in a school are recorded. Let X be the discrete random variable corresponding to the number of these pupils who have their birthday on New Year's Day.

Assuming that birthdays for the pupils constitute a random sample of size 500 from the days in the year, and that all birthdays are equally likely, find and evaluate the probability mass function for X, that is, the function f_X defined by

$$f_X(x) \equiv \mathrm{P}[X = x]$$

for appropriate values of x.

(b) Show that the probability mass function in (a) can be approximated by

$$\frac{\lambda^x}{x!}\, e^{-\lambda}$$

where $\lambda = \dfrac{500}{365} \approx 1.3699$, and verify the approximation numerically for $x = 0, 1, 2, 3, 4, 5, 6$.

[*Use the result that for large n,*

$$\left(1 - \frac{t}{n}\right)^n \approx e^{-t}$$

using the power series representation of e^t]

2. A company wishes to make two of a group of six employees, comprising three female and three male employees, redundant, by selecting two employees at random. Let X and Y be the random variables corresponding to the number of female and male employees made redundant, respectively.

Find the probability mass functions of X and Y.

3. Five balls numbered 1,2,3,4 and 5 are placed in a bag. Two balls are selected without replacement. Find the probability mass function of the following random variables:

(i) X, the largest of the two selected numbers,
(ii) Y, the sum of the two selected numbers

4. A surgical procedure is successful with probability θ. The surgery is carried out on five patients, with the success or failure of each operation independent of all other operations. Let X be the discrete random variable corresponding to the number of successful operations.

Find the probability mass function of X, and evaluate the probability that

(i) all five operations are successful, if $\theta = 0.8$,
(ii) exactly four operations are successful, if $\theta = 0.6$,
(iii) fewer than two are successful, if $\theta = 0.3$.

5. An individual repeatedly attempts to pass their driving test. Suppose that the probability that the test is passed is θ, and that the results of successive tests are independent. Let X be the discrete random variable corresponding to the number of tests taken until the individual passes.

Find the probability mass function of X, and evaluate the probability that

(i) the test is passed in three or less tests, if $\theta = 0.25$,
(ii) more than five tests are required for a pass to be obtained, if $\theta = 0.7$.

Find the cumulative distribution function of X, and check that it satisfies the required conditions for a cumulative distribution function.

6. A fair coin is tossed repeatedly, with successive tosses identical and independent. Let X be the discrete random variable corresponding to the number of tosses required to obtain three heads (that is, the sequence of tosses continues until a total of three heads has been obtained).

Find the probability mass function of X.