

M1S : EXERCISE SHEET 1

EVENTS AND SAMPLE SPACES

1. Let A , B and C be three arbitrary events. Using only the operations of union, intersection and complement, write down expressions for the following events:

- | | |
|---|--|
| (a) Only A occurs. | (b) Both A and B , but not C occurs. |
| (c) All three events occur. | (d) At least one of A , B and C occurs. |
| (e) At least two of A , B and C occur. | (f) Precisely one of A , B and C occurs. |
| (g) Precisely two of A , B and C occur. | (h) None of A , B and C occurs. |
| (i) Not more than two of A , B and C occur. | |

2. Let A , B and C be three arbitrary events. Which of the following relationships are true? Justify your answers.

- (a) $(A \cup B) \cap (A \cup C) = A \cup (B \cap C)$.
 (b) $(A \cup B) = (A \cap B') \cup B$.
 (c) $(A' \cap B) \cup (A \cap B') = (A \cup B) \cap (A \cap B)'$.
 (d) $(A \cup B)' \cap C = A' \cap B' \cap C'$.
 (e) $(A \cap B) \cap (B' \cap C) = \emptyset$.

3. A coin is tossed three times. Let A be the event that there are exactly two heads, B the event that there are more heads than tails, and C the event that the last toss is a tail. Using the operations of union, intersection and complement, find in terms of A , B and C expressions for the events:

- (a) There are more tails than heads.
 (b) There are three heads.
 (c) The first two tosses are heads.

4. Items from a production line are individually judged to be either satisfactory (S), or faulty (F). Inspection continues until two consecutive faulty items are encountered, or a total of four items have been checked, whichever occurs first.

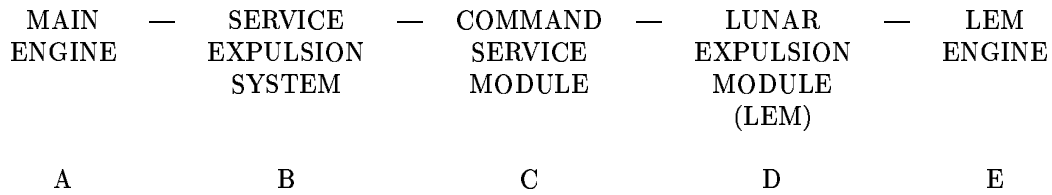
- (a) Describe the sample space for this situation
 (b) Write down the elementary outcomes in the event

“the final item inspected was satisfactory”.

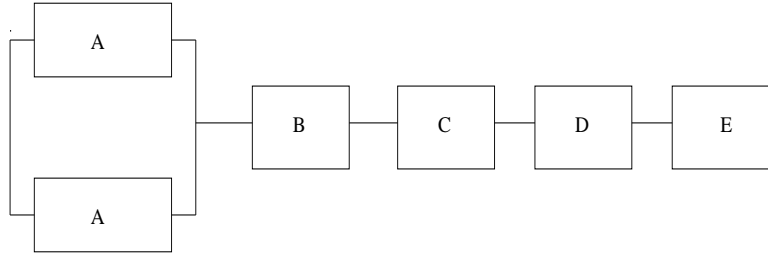
5. A box of n light bulbs contains r with broken filaments ($r < n$). Describe the sample space for the following situations: bulbs are tested, one by one, until

- (a) a defective is found;
 (b) all defectives are found.

6. A schematic diagram for a complex system, a space module, is depicted below;



Thought of as a sequence of components, which either work or do not work, and given that there is a parallel standby main engine A_2 which takes over if the initially ignited main engine A_1 fails, we have the following representation:



(a) Identifying the event that a given unit functions with corresponding letter, give a representation of the event “entire system functions”.

(b) What does this representation become if B actually consists of three components, B_1 , B_2 and B_3 , and functions *only if at least two* of these function?