M1S : ASSESSED COURSEWORK 4

Deadline : Thursday, 22nd March

Suppose that, for parameter γ $(0 < \gamma < 1)$ the pdf of a continuous random variable X with range $\mathbb{R}^+$ is to be specified as

$$f_X(x) = \gamma f_1(x) + (1-\gamma) f_2(x) \tag{1}$$

for some functions f_1 and f_2.

(a) Verify that, if f_1 and f_2 are pdfs, then f_X is a valid pdf, and show that

$$\mathrm{E}_{f_X}[X] = \gamma \mathrm{E}_{f_1}[X] + (1-\gamma)\mathrm{E}_{f_2}[X]$$

[6 MARKS]

(b) The **moment generating function** (or mgf) of a random variable with pdf f_X is denoted M_X and is defined for argument t by

$$M_X(t) = \mathrm{E}_{f_X}\left[e^{tX}\right] = \int e^{tx} f_X(x)\, dx$$

By writing out the mgf integral in full, find the mgf of random variable X having pdf given by (1) in terms of the mgfs corresponding to pdfs f_1 and f_2, denoted M_1 and M_2

[6 MARKS]

(c) The working lifetimes of two types of battery, labelled REGULAR and LONG LIFE respectively, are modelled by probability distributions f_1 and f_2

$$\text{REGULAR} \qquad f_1(x) = \frac{(2.5)^4}{\Gamma(4)} x^3 e^{-2.5x}$$

$$\text{LONG LIFE} \qquad f_2(x) = \frac{4^{11}}{\Gamma(11)} x^{10} e^{-4x}$$

where $\Gamma(.)$ is the Gamma function.

A battery is selected from a mixed box containing 80% REGULAR batteries and 20% LONG LIFE batteries, and its lifetime, X, measured. Find

(i) the expected lifetime of the selected battery, $\mathrm{E}_{f_X}[X]$

[4 MARKS]

(ii) the mgf of the lifetime of the selected battery, $M_X(t)$

[4 MARKS]