M1S : ASSESSED COURSEWORK 2

Deadline : Thursday, 15th February

In a factory employing a workforce of 200 people, a survey is to be carried out to discover whether there is an acceptable level of job satisfaction amongst the employees, who will be classified as either SATISFIED or DISSATISFIED. A pilot survey of 20 employees is embarked upon, and it is discovered that of these 20, 17 can be classified as SATISFIED and 3 as DISSATISFIED. The sample of 20 was selected from the workforce such that all such samples of the same size were equally likely to be selected.

(i) Write down a formula for the probability that the sample of size 20 would contain 17 SATISFIED employees, if the true number of SATISFIED workers in the workforce is R, for values of R in an appropriate range .

[4 MARKS]

(ii) Let A_R denote the event that the number of SATISFIED employees in the workforce is R, and let B_r denote the event that the number of SATISFIED workers in the sample of size 20 is r. Write down an expression for the conditional probability

$$\mathrm{P}\left(A_R \,|\, B_r\right).$$

State the ranges of r and R for which this conditional probability may be legitimately calculated, and explain what this conditional probability represents.

[6 MARKS]

(iii) Before the results of the survey, it can be assumed that the number of SATISFIED employees in the workforce is equally likely to take any value in the range $\{0, 1, 2, ..., 199, 200\}$. Using MAPLE, or otherwise, plot/sketch the function $f(R)$ defined by

$$f(R) = \mathrm{P}\left(A_R \,|\, B_{17}\right)$$

for appropriate values of R, and hence find the most probable value for the number of SATISFIED workers in the workforce, in light of the information gained by the survey.

[6 MARKS]

(iv) In light of the results of the survey, how probable is it that the number of SATISFIED employees exceeds 95% of the workforce ? Justify your answer, showing all relevant working.

[4 MARKS]

You may use MAPLE to find the solutions for this problem; if you do use MAPLE, attach an annotated printout of your working with your written solutions