M1S : ASSESSED COURSEWORK 1

Deadline : Friday, 2nd February

1. Three newspapers, denoted A, B and C are published in a city. A comprehensive survey indicates that amongst the adult population of the city, 20% read A, 16% read B and 14% read C, but the survey also reveals that 8% read A and B, 5% read both A and C, and 4% read B and C, and that 2% read all three newspapers. A person is selected at random from the adult population of the city (that is, all adults are equally likely to be selected). Find the probability that this person

(i) reads none of the newspapers

[2 MARKS]

(ii) reads precisely one of the newspapers

[3 MARKS]

(iii) reads at least A and B, **given** that they read at least one newspaper

[5 MARKS]

Use formal notation to express these events in terms of A, B, C.

[Hint: let Ω be the (finite) sample space comprising individuals who could be selected, and denote by A, B, C the events (subsets of Ω) that correspond to those individuals that read newspapers A, B ad C respectively. If necessary, try constructing answers using a Venn diagram with $n_\Omega = 100$ say, allocate numbers of individuals to the various parts of the diagram, then translate into a formal probability proof - the Venn diagram on its own **is not sufficient** to obtain full marks.]

2.A dice game involves rolling two fair dice (each with six equally likely scores 1,2,..,6) for which the two scores are probabilistically independent. The game is played as follows; if a combined score of 7 or 11 is obtained, the game is won; if a combined score of 2,3, or 12 is obtained, the game is lost; if any other score is obtained, the dice are rolled again repeatedly until a combined score of 7 is obtained (when the game is won) or until the combined score rolled on the very first roll is obtained (in which case the game is lost).

(i) Let E_{1j} denote the event that a combined score of j is obtained on the very first roll, for $j = 2, ..., 12$. Find $\mathrm{P}(E_{1j})$.

[2 MARKS]

(ii) Let F_n denote the event that the game is won on roll n, for $n = 2, 3, 4, ...$ By considering the partition of F_n

$$F_n \equiv \bigcup_{j=2}^{12} (F_n \cap E_{1j})$$

or otherwise, show that the probability of winning the game overall can be expressed as

$$\mathrm{P}(F_1) + \sum_{n=2}^{\infty} \sum_{j=2}^{12} \mathrm{P}(F_n \cap E_{1j}) \qquad \text{or equivalently as} \qquad \mathrm{P}(F_1) + \sum_{j=2}^{12} \sum_{n=2}^{\infty} \mathrm{P}(F_n \cap E_{1j})$$

[3 MARKS]

(iii) For each *appropriate* j find the conditional probability

$$\mathrm{P}(F_n | E_{1j})$$

and hence find the overall probability that the game is won.

[5 MARKS]

[Hint: here Ω is the (countable) set of sequences of rolls that could occur. To win on roll $n \geq 2$, given that a combined score of j was obtained on the very first roll, a sequence of $n - 2$ rolls of scores neither 7 nor j must occur next, followed finally by a score of 7 on the nth roll.]