M1S : EXERCISE SHEET 1 : SOLUTIONS

1. (a) $A \cap B' \cap C'$
   (b) $A \cap B \cap C'$
   (c) $A \cap B \cap C$
   (d) $A \cup B \cup C$
   (e) $(A \cap B \cap C') \cup (A \cap B' \cap C) \cup (A' \cap B \cap C) \cup (A \cap B \cap C)$
   (f) $(A \cap B' \cap C') \cup (A' \cap B \cap C') \cup (A' \cap B' \cap C)$
   (g) $(A \cap B \cap C') \cup (A \cap B' \cap C) \cup (A' \cap B \cap C)$
   (h) $A' \cap B' \cap C'$
   (i) $(A \cap B \cap C)' \equiv A' \cup B' \cup C'$

2. (a) TRUE : associativity
   (b) TRUE : $(A \cap B') \cup B = (B \cup A) \cap (B \cup B') = (B \cup A) \cap \Omega$.
   (c) TRUE : distributivity
   (d) FALSE : $\omega \in LHS \Longrightarrow \omega \in C$, but $\omega \in RHS \Longrightarrow \omega \in C'$ (CONTRADICTION).
   (e) TRUE : $\omega \in LHS \Longrightarrow \omega \in B$ and $\omega \in B' \Longrightarrow$ no such ω exists.

3. (a) B'
   (b) $A' \cap B$
   (c) $(A \cap C) \cup (A' \cap B)$

4. (a) $\Omega = \{FF, SFF, SSFF, FSFF, SSSS, SSSF, SSFS, SFSS, FSSS, FSFS, SFSF, FSSF\}$
   (b) $\{SSSS, SSFS, SFSS, FSSS, FSFS\}$

5. (a) $\Omega = \{A_i : i = 1, ..., n - r + 1\}$, where

   $$A_i \equiv \text{“find first defective on } i\text{th inspection”}$$

   (b) $\Omega = \{B_i : i = r, r + 1, ..., n\}$, where

   $$B_i \equiv \text{“find last defective on } i\text{th inspection”}$$

6. (a) $\Big[A_1 \cup (A_1^{'} \cap A_2)\Big] \cap B \cap C \cap D \cap E$

   (b) Replace B in (a) by the DISJOINT UNION

   $$(B_1 \cap B_2 \cap B_3^{'}) \cup (B_1 \cap B_2^{'} \cap B_3) \cup (B_1^{'} \cap B_2 \cap B_3) \cup (B_1 \cap B_2 \cap B_3)$$

NOTE: Can use Venn Diagrams as an aid to solving these problems

TUTORIAL SHEET WEEK 2 : SOLUTIONS

1. (a) $\Omega = \{FF, FM, MF, MM\}$

(b) $A = \{FF\}$, $B = \{MM\}$, $C = \{FM, MF, MM\}$

$\Longrightarrow$ $A \cap B = \emptyset$
$A \cup B = \{FF, MM\}$
$A \cap C = \emptyset$
$A \cup C = \Omega$
$B \cap C = \{MM\}$
$B \cup C = C$
$B' \cup C = \{FM, MF\}$

2. Events are found by inspection of sample outcomes in Ω (form Ω as a two-way, 6×6 table whose entries are ordered pairs of scores). Can list elements in each case :

$$\begin{aligned}
A &= \{(1,2),(2,2),(3,2),(4,2),(5,2),(6,2),\\
&\quad (1,4),(2,4),(3,4),(4,4),(5,4),(6,4),\\
&\quad (1,6),(2,6),(3,6),(4,6),(5,6),(6,6)\}\\
C' &= \{(2,2),(2,4),(2,6),(4,2),(4,4),(4,6),(6,2),(6,4),(6,6)\}\\
A \cap B &= \{(2,2),(4,2),(6,2),(2,4),(4,4),(6,4),(2,6),(4,6),(6,6)\}\\
A \cap B' &= \{(1,2),(3,2),(5,2),(1,4),(3,4),(5,4),(1,6),(3,6),(5,6)\}\\
A' \cup B &= \Omega \setminus \{(1,2),(1,4),(1,6),(3,2),(3,4),(3,6),(5,2),(5,4),(5,6)\}\\
A' \cap C &= \{(1,1),(2,1),(3,1),(4,1),(5,1),(6,1),\\
&\quad (1,3),(2,3),(3,3),(4,3),(5,3),(6,3),\\
&\quad (1,5),(2,5),(3,5),(4,5),(5,5),(6,5)\}
\end{aligned}$$

and the required events have 18, 9, 9, 9, 27 and 18 elements respectively.

3. Label defectives D_1, D_2, and others G_1, G_2, G_3. Then Ω is the set

$$\{(D_1,D_2),(D_1,G_1),(D_1,G_2),(D_1,G_3),(D_2,G_1),(D_2,G_2),(D_2,G_3)(G_1,G_2),(G_1,G_3),(G_2,G_3)\}$$

and

$$A = \{(G_1,G_2),(G_1,G_3),(G_2,G_3)\}$$