M1S TUTORIAL SHEET : WEEK 8

CONTINUOUS RANDOM VARIABLES

Recall that a random variable X is a function that maps a sample outcome ω in the sample space Ω of an experiment to a real number x,

$$\begin{aligned} X: \quad \Omega &\longrightarrow \mathbb{X} \subseteq \mathbb{R} \\ \omega &\longmapsto x \end{aligned}$$

and $X(\omega) = x$, and $\mathbb{X}$ is the range of X.

When Ω and $\mathbb{X}$ are *countable* sets, the random variable X is termed **discrete**.

However, for some experiments, the sets Ω and $\mathbb{X}$ are *uncountable*, and there is a continuum of possible outcomes to the experiment; thus situation arises, for example, in *measurement* experiments.

We often require a probability specification to reflect this continuum of possible values of X, and typically this requirement is met by ensuring that the probability content of an interval of $\mathbb{X}$ increases smoothly as the interval size increases.

In this case, the random variable X is termed **continuous**, and the range $\mathbb{X}$ is an interval (or the union of intervals) in $\mathbb{R}$. Consequently, an event $E \subseteq \Omega$ is equivalent to the event "$X \in \mathbb{X}_E$" for some interval (or union of intervals) $\mathbb{X}_E \subseteq \mathbb{R}$, and

$$\mathrm{P}(E) \quad \equiv \quad \mathrm{P}[\ X \in \mathbb{X}_E\ ]$$

The specification of the **probability distribution** of X is achieved, as in the discrete case, via the **cumulative distribution function**, again denoted F_X and defined by

$$F_X(x) \equiv \mathrm{P}[X \leq x]$$

for real-values of x. In practically all cases, the probability distribution can also specified via the **probability density function**, which is denoted (by analogy with the discrete case) f_X, and is defined implicitly by

$$F_X(x) = \int_{-\infty}^{x} f_X(t)\ dt$$

and explicitly, using the fundamental result of calculus, by

$$f_X(x) = \frac{d}{dt}\left\{F_X(t)\right\}_{t=x}$$

for real-values of x, that is, $f_X(x)$ is the first derivative of F_X, evaluated at x.