M1S TUTORIAL SHEET : WEEK 7

Random variables provide a mathematically convenient means of describing the outcomes of experiments. A random variable X is merely a function that maps a sample outcome ω in the sample space Ω of an experiment to a real number x, that is

$$\begin{aligned} X: \quad \Omega &\longrightarrow \mathbb{R} \\ \omega &\longmapsto x \end{aligned}$$

and $X(\omega) = x$. Usually, X maps Ω only onto a subset of $\mathbb{R}$; this subset is the **range** of the random variable, and can be denoted $\mathbb{X}$.

An event $E \subseteq \Omega$, which is a *collection* of sample outcomes, is mapped to a *collection* of real-numbers, $\mathbb{X}_E$, say.

A special case of this type of mapping arises when Ω is of the form

$$\Omega = \{\omega_1, \omega_2, \omega_3, ...\}$$

that is, when Ω is a *countable* set. In this case, the range of random variable $\mathbb{X}$ is also countable and

$$\mathbb{X} = \{x_1, x_2, x_3, ...\}\ .$$

In this situation, the random variable is termed **discrete**.

For any event E, by construction, we must have that

$$\mathrm{P}(E) \quad \equiv \quad \mathrm{P}[\ X \in \mathbb{X}_E\ ]$$

and hence can shift attention to specification of the probability on the right-hand side, that is the **probability distribution** of X. This specification is typically achieved via the **probability mass function** which is denoted f_X, and is defined by

$$f_X(x) \equiv \mathrm{P}[X = x]$$

for real-values of x; note that $f_X(x)$ is automatically **zero** if $x \notin \mathbb{X}$.

An alternative method of specifying the probability distribution of X is the **cumulative distribution function**, which is denoted F_X, and defined by

$$F_X(x) \equiv \mathrm{P}[X \leq x]$$

for real-values of x.