M1S TUTORIAL SHEET : WEEK 4

The following example is an illlustration of concepts such as conditional probability, independence and conditional independence, the general multiplication rule, and the fundamental probability theorems (The Theorem of Total Probability and Bayes Theorem).

A shop sells fuses produced by three manufacturers; each manufacturer supplies a deluxe and a standard type of fuse. A mixed batch of 500 fuses sold, and the number of faulty fuses of each type and for each manufacturer is recorded. By considering the following events; $M_i \equiv$ "fuse produced by manufacturer i" for $i = 1, 2, 3$, $D \equiv$ "Deluxe type of fuse" and $F \equiv$ "Fuse Faulty", a summary of the data can be presented as a 3-way table

	M_1		M_2		M_3	
	D	D'	D	D'	D	D'
F	20	16	30	20	15	10
F'	100	64	120	30	60	15

so that, for example, the number of deluxe fuses from manufacturer 1 that are faulty is 20, whereas the number of standard fuses from manufacturer 1 that are faulty is 16, etc.

(a) A fuse is selected with equal probability from the 500. What is the probability that

(i) it is faulty ?

(ii) it was produced by manufacturer 1 ?

(b) Given that the selected fuse is faulty, what is the conditional probability that

(i) it is a deluxe fuse ?

(ii) it is a fuse produced by manufacturer 1 ?

(iii) it is a deluxe fuse produced by manufacturer 1 ?

(c) Describe, evaluate, and comment on the following conditional probabilities:

(i) $\mathrm{P}(F \mid M_1)$, $\mathrm{P}(F \mid M_2)$, $\mathrm{P}(F \mid M_3)$

(ii) $\mathrm{P}(F \mid D)$, $\mathrm{P}(F \mid D')$

(iii) $\mathrm{P}(F \mid M_1 \cap D)$, $\mathrm{P}(F \mid M_2 \cap D)$, $\mathrm{P}(F \mid M_3 \cap D)$.

(iv) $\mathrm{P}(F \mid M_1 \cap D')$, $\mathrm{P}(F \mid M_2 \cap D')$, $\mathrm{P}(F \mid M_3 \cap D')$.