# M1S : EXERCISES 5

## OCCUPANCY PROBLEMS AND HYPERGEOMETRIC PROBABILITIES

**ASSESSED COURSEWORK**
(Deadline : Monday 21st February)

(a) A bag contains $b$ black balls and $r$ red balls. If balls are selected from the bag without replacement, and all balls are equally likely to be selected from those remaining for each successive selection, show that the probability that the probability that the first red ball is selected on the $(k+1)$st draw is

$$p(k) = \frac{\binom{r+b-k-1}{r-1}}{\binom{r+b}{b}}$$

for $k = 0, 1, ..., b$.

[*Hint: consider a binary sequence representation.*]

[*5 MARKS*]

Find the probability that the last ball drawn is red.

[*3 MARKS*]

(b) $r = 8$ indistinguishable balls are to be allocated to $n = 6$ boxes, with all allocations equally likely. Find the probability that the first two boxes contain at most four balls in total.

[*Hint: form a partition of the event of interest by considering the events that the first two boxes contain precisely $k = 0, 1, 2, 3, 4$ balls in total. Then enumerate the ways of allocating $k$ balls to two boxes and $8 - k$ balls to four boxes simultaneously.*]

[*6 MARKS*]

(c) A committee of $n = 5$ students is to be selected, supposedly at random, from a class of $N = 200$ comprising $R = 120$ males and $N - R = 80$ females.

Comment (using probability reasoning only ...) on the evidence for sex-bias in the selection process if the committee comprises $r = 5$ males, and no females.

[*Hint: evaluate the probability of the observed event assuming that the selection process is truly random, that is, that all selections of $n = 5$ from $N = 200$ are equally likely.*]

[*6 MARKS*]

**EXERCISES** (not to be handed in)

1. In the National Lottery, six winning numbers are drawn from forty nine. Suppose that one ticket, with a single selection of six numbers, is purchased for each draw.

(i) What is the probability that, on any week, the ticket contains no winning numbers ?

(ii) Assuming that draws on successive weeks are independent, find an expression in $x$ for the probability that the tickets purchased purchased contain no winning numbers for $x$ successive weeks, where $x = 1, 2, 3.....$

2. The game of *Keno* is a lottery-type game which consists of cards numbered 1,2,...,80, twenty of which are to be drawn as winning cards. A player attempts to win at the game by nominating $n$ cards before the draw is made, where $n$ is any number not greater than fifteen. The amount won depends on the number of nominated cards that are subsequently drawn as winning cards.

What is the probability that no nominated cards are drawn as winning cards

(i) if $n = 5$ ?

(ii) if $n = 10$ ?

(iii) if $n = 15$ ?

3. If $n$ balls are placed at random into $n$ boxes, what is the probability that precisely one box remains empty ?

4. Each of $n$ sticks is broken into two parts, long and short, and a new set of $n$ sticks formed by pairing and joining the $2n$ parts at random. What is the probability that

(i) each stick is paired and rejoined into its original form that is, there is a match between the rejoined long and short parts for all $n$ sticks.

(ii) each of the $n$ long parts are rejoined with a short part.

5. A class comprises 30 pupils. What is the probability that, among the twelve months, six contain two of the birthdays, and six contain three of the birthdays of the pupils ?

6. A group of $2n$ females and $2n$ males is randomly divided into two subgroups. What is the probability that each subgroup contains $n$ females and $n$ males ?

7. Two hands of thirteen cards are dealt (without replacement) from an ordinary pack. What is the probability that one hand contains exactly $n$ Hearts, and the other contains exactly $m$ Hearts ?

Now suppose that four hands of thirteen cards are dealt (without replacement) from an ordinary pack. What is the probability that the four hands contain $r_1, r_2, r_3, r_4$ Hearts respectively, for non-negative integers satisfying $r_1 + r_2 + r_3 + r_4 = 13$ ?