CHEM. ENG. II - PROBABILITY AND STATISTICS
ASSESSED COURSEWORK

(**Deadline : Friday 23rd March 2001:** Questions carry 20 Marks each)

1. Purity tests are to be carried out on a product. Each sample is to be tested twice, and is only passed for sale if **both** tests are passed. Let events T_1 and T_2 correspond respectively to the events that first and second tests are passed, and let S be the event that, in fact, the sample is of suitable quality..

Suppose that the test used is quite accurate but fallible, in that it detects impurity in an impure sample in 99 % of tests, and indicates purity for pure samples in 95 % of tests.

The rate of production of impure samples is can be quantified by rigorous quality assurance; it can be guaranteed that, on average, only 5 samples in 1000 are impure.

If it is assumed the results of the two tests are *conditionally independent* given the purity or impurity of the samples, give expressions for, and evaluate

(i) the probability that the first test is passed

(ii) the conditional probability that the sample is impure, given that the first test is passed.

(iii) the probability that both tests are passed, so that the sample is cleared for sale.

(iv) the conditional probability that the sample is impure, given that both tests are passed.

If a batch of 20 samples are tested in this fashion, and passed for sale, what is the probability distribution of the number of samples (out of 20) that are actually impure ? Give a full specification for the distribution (parameter values etc).

For events A, B and C, with $\mathrm{P}(C) > 0$, A and B are *conditionally independent* given C if

$$\mathrm{P}(A \cap B|C) = \mathrm{P}(A|C)\mathrm{P}(B|C)$$

2. A probability distribution that has been found to be suitable for modelling the amount of outflow (in gallons per minute gpm) from a waste-pipe is specified by a (continuous) probability density function (pdf) f_X which takes the form

$$f_X(x) = \frac{c}{(2+x)^{\alpha+1}} \qquad x > 0$$

and zero otherwise. The accuracy or validity of this model is to be tested by observing data.

(i) Find the value of constant c (as a function of α)

(ii) Find the corresponding cumulative distribuion function (cdf), F_X

(iii) Use your answer from (ii) to evaluate the probability that the outflow exceeds 8 gpm if $\alpha = 2$.

The outflow was monitored, and six observations of flow rate made; the observations were

0.709 16.261 0.374 0.912 0.499 1.263

Find the **maximum likelihood estimate** of α derived from this sample

3. The interior surfaces of a cylinder housing a manufacturing process are insulated, as the efficiency of the process is critically influenced by the experimental operating temperature, which must be maintained as constant as possible at a specified target level, 1500C. Two different coating materials are used for insulation, and the operating temperatures for a number of different runs of the process are recorded (in C)

COATING A	1479.97	1479.69	1513.99	1511.31	1495.58		
COATING B	1482.72	1497.82	1469.54	1480.31	1467.59	1457.58	1501.02

Assuming the all the data are normally distributed, using formal quantitative statistical testing procedures, find and discuss any evidence that

(i) the two coatings are, individually, achieving their insulation goal

(ii) there is no difference between the effectiveness of the two coatings