CHEM. ENG. II - PROBABILITY AND STATISTICS

EXERCISES 5

1. The following data are believed to be a random sample from a $N(\mu, \sigma^2)$ distribution

$$249, 254, 243, 268, 253, 269, 287, 241, 273, 306$$
$$303, 280, 260, 256, 278, 344, 304, 283, 310.$$

Test the hypothesis

$$\begin{aligned} H_0 &: \sigma = 30.0 \\ H_1 &: \sigma \neq 30.0 \end{aligned}$$

at the 5 % significance level. Construct a 95 % confidence interval for μ

(i) σ is **known**, $\sigma = 30.0$,

(ii) σ is **unknown**.

and a 95 % confidence interval for σ^2.

2. It can be shown that if U and V are independent Chi-squared random variables with ν_1 and ν_2 degrees of freedom respectively, then the random variable Q defined by

$$Q = \frac{U/\nu_1}{V/\nu_2}$$

has an *F-distribution* with ν_1 and ν_2 degrees of freedom (this distribution forms the basis of the F-test for equal variances). However, statistical tables for the F-distribution only include values in the right tail of the distribution, that is, the value of x which satisfies

$$F_{F_{\nu_1,\nu_2}}(x) = 0.975.$$

where $F_{F_{\nu_1,\nu_2}}(.)$ is the cdf of the F-distribution.

Show that

$$F_{F_{\nu_1,\nu_2}}(x) = 0.975 \Longleftrightarrow F_{F_{\nu_2,\nu_1}}(1/x) = 0.025$$

hence values for the left tail of the F-distribution can be calculated directly from the tables.

3. The following data are believed to be random samples from two normal distributions, $N(\mu_1, \sigma_1^2)$ and $N(\mu_2, \sigma_2^2)$;

SAMPLE 1	:	$150, 142, 160, 110, 120, 240, 152, 136, 122, 200, 160, 102, 152, 208$
SAMPLE 2	:	$70, 110, 85, 90, 210$

(i) Carry out an appropriate two-sample hypothesis test of $\mu_1 = \mu_2$, if it is assumed that $\sigma_1 = \sigma_2$.

(ii) Is there any evidence that $\sigma_1 = \sigma_2$?

(iii) Construct a 95 % confidence interval for $\mu_1 - \mu_2$.

(iv) Construct a 95 % confidence interval for σ_1/σ_2.

4. Continuous random variable X has a *Weibull* distribution, so that its cdf, F_X, is given by

$$F_X(x) = \mathrm{P}[X \leq x] = 1 - \exp\{-\lambda x^\alpha\} \qquad x \geq 0$$

for parameters $\alpha, \lambda > 0$.

Suppose that a sample of data $x_1, ..., x_n$ is available. By considering the solutions $q_1, ..., q_n$ to equations

$$F_X(q_i) = \frac{i}{n+1} \qquad i = 1, ..., n$$

describe how to check that the Weibull model is an appropriate probability model for the sample using a probability plotting approach.

5. A sample of count data was obtained, and a frequency table was formed. The data are summarized below.

Count	Frequency
0	44
1	86
2	56
3	12
4	2

Using the Chi-squared Goodness of fit test, carry out hypothesis tests of whether the following probability models are appropriate for the data:

(i) a uniform distribution on the set $\{0, 1, 2, 3, 4\}$

(ii) a binomial distribution with parameters $n = 4$, $p = 0.25$.

(iii) a binomial distribution with parameter $n = 4$ and p unspecified

(iv) a Poisson distribution with parameter $\lambda = 1.0$

(v) a Poisson distribution with parameter λ unspecified.