CHEM. ENG. II - PROBABILITY AND STATISTICS

EXERCISES 1

1. Suppose that one ball is to be selected from a bag containing ten red balls numbered 1 to 10, and ten blue balls numbered 1 to 10. Let A be the event that an even numbered ball is selected, B be the event that a blue ball is selected, and C be the event that a ball with a number less that five is selected. Describe the sample space S, and describe the following events in words, and calculate their probability;

(1) $A \cap B \cap C$
(2) $B \cap C^{'}$
(3) $A \cup B \cup C$
(4) $A \cap (B \cup C)$
(5) $A^{'} \cap B^{'} \cap C^{'}$

2. A sample of components are tested for two qualities E and F. The following table summarizes the number of components in the sample that had both, one or other, or neither of the qualities.

	E	$E^{'}$
F	6	12
$F^{'}$	2	4

Calculate the probability that, if a component is selected at random from the sample it has

(1) quality E
(2) quality F

Now, assuming when it is selected, the component is known to have quality E. Calculate the probability that it has quality F. Repeat the calculation if it is known that the component does not have quality E.

Repeat each calculation for the following table.

	E	$E^{'}$
F	6	12
$F^{'}$	4	6

For both tables, validate the relationships between joint and conditional probabilities (that is, that, for events E and F, $\mathrm{P}(E \cap F) = \mathrm{P}(E)\mathrm{P}(F|E)$), and see whether the two events involved are independent.

3. An electronic circuit comprises five components A,B,C,D and E installed in series. The circuit functions only if each of the components function. If F represents the event that the circuit functions, and A the event that component A functions etc., give an expression for $\mathrm{P}(F)$ in terms of the probability of the intersection of the events A, B, C, D, E. Using the chain rule, rewrite this probability in terms of conditional probabilities.

Suppose that $\mathrm{P}(A) = \mathrm{P}(B) = \mathrm{P}(C) = \mathrm{P}(D) = \mathrm{P}(E) = 0.9$, with the components functioning (or not functioning) independently. Calculate $\mathrm{P}(F)$.

Suppose now that $\mathrm{P}(B|A) = 0.95$, but $\mathrm{P}(B|A^{'}) = 0.75$, with all other components functioning independently. Using the Theorem of Total Probability, calculate $\mathrm{P}(B)$.

Finally, suppose that A and B are installed in parallel with each other, but in series with the other components, and that the circuit functions if either A or B or both function, and C,D and E function. Calculate $\mathrm{P}(F)$ if
(i) A and B are independent
(ii) $\mathrm{P}(B|A) = 0.95$, $\mathrm{P}(B|A^{'}) = 0.75$.

4. A person either suffers (event S) or does not suffer ($S^{'}$) from a particular disease. There is a diagnostic test for the disease that is either positive (event T) or negative ($T^{'}$). Suppose that $\mathrm{P}(S) = 0.001$ (the disease is rare), and given that the person is a sufferer from the disease, the diagnostic test is positive (gives a correct diagnosis) with probability 1.0 (so that $\mathrm{P}(T|S) = 1.0$). Suppose, however, that the test is positive if the person is **not** a sufferer (i.e. gives a false positive) with probability 0.1 (so that $\mathrm{P}(T|S^{'}) = 0.1$).
Calculate the probability that

(i) the test gives a positive result $\mathrm{P}(T)$,
(ii) the test gives an incorrect diagnosis,
(iii) the person is a sufferer **given** that the test result is positive.

5. Consider two coins, of which one is normal and the other has a Head on both sides. One of the coins is selected with equal probability $1/2$, and tossed repeatedly, with tosses considered independent.

Evaluate the conditional probability that the selected coin is normal, **given** that the first n tosses are Heads.

Now consider two coins, of which one is normal and the other is biased so that the probability of obtaining a Head on a single toss is $p > 1/2$. Again, one of the coins is selected with equal probability, and tossed n times independently. Let E be the event that the n tosses result in k Heads and $n - k$ Tails, and let F be the event that the coin is fair. Find expressions for

(i) $\mathrm{P}(E)$

(ii) $\mathrm{P}(F|E)$

Note: There are

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

sequences of tosses that contain precisely k Heads and $n - k$ Tails in some arrangement.

6. The probability that a tree has n flowers is given by $(1-p)p^n$ for $n = 0, 1, 2, ...$ Each flower has probability $2/3$ of being pollinated and producing fruit, and each fruit has probability of $1/4$ of not ripening fully. Deduce that the probability of a flower producing a ripe fruit is $1/2$. **Given** that a tree bears r ripe fruit, calculate the conditional probability that it originally had n flowers using the general version of Bayes Theorem.

Note: The Negative Binomial expansion takes the form

$$\frac{1}{(1-x)^n} = \sum_{i=0}^{\infty} \binom{n+i-1}{i} x^i$$

for $|x| < 1$.

7. In a game show, a competitor is asked to select one of three doors, behind only one of which is a prize. After the selection has been made, the gameshow host opens one of the two remaining doors, revealing no prize. The host then asks the competitor whether they want to switch their choice to the other unopened door, or whether they want to stick to their original choice.

Probabilistically, is it advantageous for the competitor to switch ?