The Natural Number Game : an introduction to Lean tactics

K. Buzzard

13th July 2020, LFTCM2020

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- You will have Lean and `leanproject` installed correctly.

Note: if you know all this stuff already, Rob Lewis is running a parallel thread on metaprogramming (i.e. tactic-writing).

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I made the natural number game with Mohammad Pedramfar.

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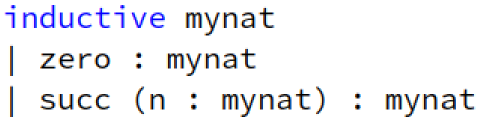

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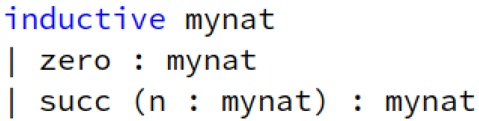

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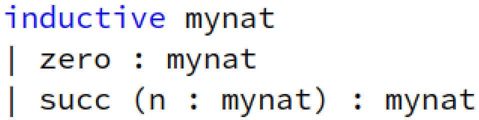

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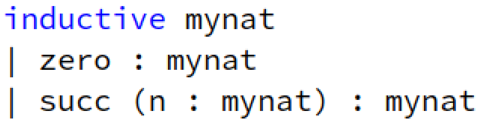

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- A new type called `mynat`;
- New terms `mynat.zero` and `mynat.succ`;
- A recursor `mynat.rec`.

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All proofs are written in tactic mode (within a `begin`/`end` block).

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If you have finished the natural number game, then try the **lost levels.** Install the natural number game locally using `leanproject`, open the project (open *folder*), open the file `src/game/world10/level18q.lean`, and fill in the `sorry`s!

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If you don't understand anything – ask! Zoom, Zulip etc.