Question 3.

(a) Say P and Q are true/false statements, and we know

$$(P \Rightarrow Q) \wedge (Q \Rightarrow \neg P) \wedge (\neg P \Rightarrow \neg Q).$$

Is Q true or false? Give your reasoning.

(b) Are the following statements about a real number x true or false? Give the briefest of justifications.

i) $x = 1 \Rightarrow x^2 = 1$;

ii) $x = 1 \Rightarrow (x = 1 \vee x = -1)$.

(c) Define $A = \{1, 2, 3\}$ and $B = \{A\}$ and $C = \{B\}$. Are the following statements true or false? Just write T or F.

i) $1 \in A$;

ii) $\{1\} \in B$;

iii) $A \in C$.

Answer.

(a) We are given that $P \Rightarrow Q$ and $Q \Rightarrow \neg P$ and $\neg P \Rightarrow \neg Q$.

Let's assume P is true. Then $P \Rightarrow Q$, so Q is true, and then $Q \Rightarrow \neg P$, so $\neg P$ is true, thus P is false. A contradiction!

Hence P must be false, meaning $\neg P$ is true, and we are told $\neg P \Rightarrow \neg Q$, so $\neg Q$ must be true as well, giving that Q is false. **(3 marks)**

(b) i) This is true – square both sides.**(2 marks)**

ii) This is true – if the left hand side is true then $x = 1$ so the right hand side is true. Alternatively one can argue that this part is logically equivalent to the part before.**(2 marks)**

(c) TFF**(3 marks)**

Notes to markers: there are several ways to argue for some of these, so use common sense and please don't deduct marks if students argue in a different, but logically correct, manner (otherwise they will complain). If in doubt I (Kevin Buzzard) will happily field questions (just email me and I'll try to respond promptly).