

Question 4.

Calculate the number of elements of the set

$$S = \{k \in \mathbb{N}, 0 \leq k < 149 \mid \text{hcf}(k, 360) = 1\}$$

(justify your answer).

Answer.

Note that $\text{hcf}(c, 360) = 1$ if and only if c is coprime with 2, 3, 5.

Let $\Omega = \{k \in \mathbb{N} \mid 0 \leq k < 149\}$ and denote by $S_d \subset \Omega$ the subset of numbers that are divisible by d , we have by inclusion-exclusion:

$$\begin{aligned} |S| &= |\Omega| - |S_2| - |S_3| - |S_5| + |S_6| + |S_{10}| + |S_{15}| - |S_{30}| = \\ &= 149 - \left\lceil \frac{149}{2} \right\rceil - \left\lceil \frac{149}{3} \right\rceil - \left\lceil \frac{149}{5} \right\rceil + \left\lceil \frac{149}{6} \right\rceil + \left\lceil \frac{149}{10} \right\rceil + \left\lceil \frac{149}{15} \right\rceil - \left\lceil \frac{149}{30} \right\rceil = \\ &= 149 - 75 - 50 - 30 + 25 + 15 + 10 - 5 = 39 \end{aligned}$$

(5 marks) for correctly setting up the problem using inclusion-exclusion and **(5 marks)** for the arithmetic.