Question 1.

(a) Give example of a relation that is:

(i) reflexive and symmetric but not transitive;

(ii) reflexive and transitive but not symmetric;

Can you find a relation that is symmetric and transitive, but not reflexive?

(b) Compute the smallest residue of 3^{51} mod 17.

Answer.

(a) For (i) take $S = \{1,2,3\}$ and $R = \{(1,1),(2,2),(3,3),(1,2),(2,1),(2,3),(3,2)\}$. **(2 marks)**

For (ii) take $S = \mathbb{R}$ and $R = \{(x,y) \in \mathbb{R} \times \mathbb{R} \mid x \leq y\}$. **(2 marks)**

The answer to the last question is yes: take $S = \{\emptyset\}$ and $R = \emptyset \subset S \times S$: R is symmetric and transitive but not reflexive. **(2 marks)**

(b) We have that $51 = 3 \times 16 + 3$ so by Fermat's little theorem (implying that $3^{16} \equiv 1$ mod 17):

$$3^{51} \equiv (3^{16})^3 3^3 \equiv 3^3 \equiv 27 \equiv 11$$

mod 17. Since $0 \leq 11 < 17$, 11 is the smallest residue of 3^{51} mod 17. **(4 marks)**