Question 4.

(a) Compute $c = \text{hcf}\,(351, 63)$ and find $x,\ y \in \mathbb{Z}$ such that $c = 351x + 63y$.

(b) For all $n \in \mathbb{Z}$, compute $\text{hcf}\,(2n + 2, 3n + 2)$. Justify your answer.

Answer.

(a) First, $351 = 9 \times 39$ and $63 = 9 \times 7$, hence $\text{hcf}\,(351, 63) = 9\text{hcf}\,(39, 7) = 9$. **(2 marks)** Now $39 = 5 \times 7 + 4$, so we have:

$$1 = 2 \times 4 - 7 = 2 \times (39 - 5 \times 7) - 7 = 2 \times 39 - 11 \times 7$$

Therefore $9 = 351x + 63y$ with $x = 2$, $y = -11$.[1] **(3 marks)**

(b) **(5 marks)** Let $d = \text{hcf}\,(2n + 2, 3n + 2)$. From the identity

$$3(2n + 2) - 2(3n + 2) = 2$$

we conclude that $d = 1$ or 2.[2] Now $2n+2$ is always even so $d = 2$ if and only if $3n+2$ is also even, that is, if and only if n is even. The final answer is

$$d = \begin{cases} 1 \text{ if } n \text{ is odd} \\ 2 \text{ if } n \text{ is even} \end{cases}$$

[1] There are of course other ways to answer this question, e.g. take the pair 351, 63 head-on with Euclid. All methods are fine if executed correctly.

[2] Perhaps students who get this far should be given 2 marks.