Name (IN CAPITAL LETTERS!): .... **TID:**

CID: .... **Personal tutor:** ....

Question 3.

(a) Is the following argument valid? If it IS valid, write it out as a string of statements connected by $\Rightarrow$:

If a movie is not worth seeing, then it was not made in England. A movie is worth seeing only if the critic Martin Filmgoer reviews it. The movie *The mathematical games* was not reviewed by Martin Filmgoer. Therefore *The mathematical games* was not made in England.

(b) Describe the following sets and carefully justify your answers:

$$\bigcup_{n=1}^{\infty}[n,\infty) \quad \text{and} \quad \bigcap_{n=1}^{\infty}[n,\infty)$$

Answer.

(a) [1] Let us denote the statements by letters as follows: **5 marks**

W(X) movie X is worth seeing;

E(X) movie X is made in England;

MR(X) Martin Filmgoer reviews movie X.

The first two statements are clearly intended to be taken as axioms of the theory:

Axiom 1 $\forall X$ a movie, $\overline{W(X)} \Rightarrow \overline{E(X)}$ (equivalently: $\forall X$ a movie, $E(X) \Rightarrow W(X)$);

Axiom 2 $\forall X$ a movie, $W(X) \Rightarrow MR(X)$ (equivalently: $\forall X$ a movie, $\overline{MR(X)} \Rightarrow \overline{W(X)}$).

Next we are told for a fact that for $X =$ *The mathematical games*, $\overline{MR(X)}$. What can we conclude from this?

[1] For educational purposes, I spell this solution out in far greater detail than needed. For the marker: please decide a reasonable mark scheme and apply it consistently.

We can of course "concatenate" the two axioms:

$$(\forall X \text{ a movie, } \overline{MR(X)} \Rightarrow \overline{W(X)})\&(\forall X \text{ a movie, } \overline{W(X)} \Rightarrow \overline{E(X)}) \Rightarrow \\ (\forall X \text{ a movie, } \overline{MR(X)} \Rightarrow \overline{E(X)})$$

In other words we have a **Theorem:** $\forall X$ a movie, $\overline{MR(X)} \Rightarrow \overline{E(X)}$.

The conclusion is trivial at this point, but let me spell it out in painful detail, for it follow a classic 3-step "syllogistic" scheme:

(i) $\forall x \in A$, $P(X)$;

(ii) $a \in A$;

(iii) Therefore, $P(a)$.

In our case this amounts to:

(i) $\forall X$ a movie, $\overline{MR(X)} \Rightarrow \overline{E(X)}$;

(ii) *The mathematical games* is a movie;

(iii) Therefore, $\overline{MR(\textit{The mathematical games})} \Rightarrow \overline{E(\textit{The mathematical games})}$;

Now we are *assuming* $\overline{MR(\textit{The mathematical games})}$, hence we conclude from *(iii)* that $\overline{E(\textit{The mathematical games})}$: in other words, *The mathematical games* is not made in England. The argument is indeed valid.

(b) First,

$$\bigcup_{n=1}^{\infty}[n,\infty) = [1,\infty)$$

indeed only $\subset$ requires proof. Suppose $x \in \bigcup_{n=1}^{\infty}[n,\infty)$. Then by definition $\exists n \in \mathbb{N}$ such that $x \in [n,\infty)$, but then, as $1 \leq n$ implies $[1,\infty) \subset [n,\infty)$, we have that $x \in [1,\infty)$ is also true. **2 marks**

Second,

$$\bigcap_{n=1}^{\infty}[n,\infty) = \emptyset$$

We will derive a contradiction from the mere assumption that there is an element $x \in \bigcap_{n=1}^{\infty}[n,\infty)$. Indeed we would have that $\forall\, n \in \mathbb{N}$, $n \leq x$. This last statement is *exactly* $\overline{P}$, where P is the Archimedean axiom. This is a contradiction. **3 marks**