Name (IN CAPITAL LETTERS!): .... **TID:** ☐

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Question 2. Use De Moivre's formula for complex numbers to prove the following trigonometric identities:

$$\cos 4\theta = 8\cos^4\theta - 8\cos^2\theta + 1\,,$$
$$\sin 4\theta = 4\cos^3\theta\sin\theta - 4\cos\theta\sin^3\theta\,.$$

Answer. 10 marks

$$\begin{aligned}\cos 4\theta + i\sin 4\theta &= (\cos\theta + i\sin\theta)^4 = \\ &= cos^4\theta - 6\cos^2\theta\sin^2\theta + \sin^4\theta + i(4\cos^3\theta\sin\theta - 4\cos\theta\sin^3\theta)\end{aligned}$$

From this, equating real and imaginary parts, we get:

$$\begin{aligned}\cos 4\theta &= \cos^4\theta - 6\cos^2\theta\sin^2\theta + \sin^4\theta = \\ &= \cos^4\theta - 6\cos^2\theta(1-\cos^2\theta) + (1-\cos^2\theta)^2 = \\ &= 8\cos^4\theta - 8\cos^2\theta + 1\end{aligned}$$

and

$$\sin 4\theta = 4\cos^3\theta\sin\theta - 4\cos\theta\sin^3\theta$$