Name (IN CAPITAL LETTERS!): ..... **TID:** ☐

CID: ..... **Personal tutor:** .....

Question 4.

In this question a, b, m, n are all in $\mathbb{N}\setminus\{0\}$. For each of the following statements, give a proof or exhibit a counterexample.

(a) If $a^n|b^n$, then $a|b$.

(b) If $n^n|m^m$, then $n|m$.

(c) If $a^n|2b^n$ and $n>1$ then $a|b$.

Answer.

(a) The statement is true. Let $a=\prod p_i^{r_i}$, $b=\prod p_i^{s_i}$ be the prime decompositions of a, b. If $nr_i \le ns_i$ for all i, then $r_i \le s_i$ for all i. **(2 marks)**

(b) The statement is false. For a counterexample consider $n=4$, $m=10$. It is clear that $4^4=2^8|10^{10}=2^{10}5^{10}$. **(4 marks)**

(c) The statement is true. Let

$$a=2^r\prod p_i^{r_i}, \quad b=2^s\prod p_i^{s_i}$$

be the prime decompositions of a, b (where it is understood that all $p_i\neq 2$). The assumptions mean $nr_i\le ns_i$ for all i, and $nr\le ns+1$. It is clear that $r_i\le s_i$, but also $r\le s+\frac{1}{n}$ implies (for $n>1$) that $r\le s$. **(4 marks)**