Name (IN CAPITAL LETTERS!): .... **TID:**

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Question 2. (10 marks) Factorize the polynomial x^4+1 into a product of linear and quadratic polynomials with *real* coefficients.

Answer. First solve the equation $x^4+1=0$ for $x \in \mathbb{C}$. The solutions are

$$x_1 = e^{i\frac{\pi}{4}}, \quad x_2 = e^{i\frac{3\pi}{4}}, \quad x_3 = e^{-i\frac{3\pi}{4}}, \quad x_4 = e^{-i\frac{\pi}{4}}.$$

The factors of x^4+1 are the two polynomials

$$(x - e^{i\frac{\pi}{4}})(x - e^{-i\frac{\pi}{4}}) = \left(x - \frac{\sqrt{2}+i\sqrt{2}}{2}\right)\left(x - \frac{\sqrt{2}-i\sqrt{2}}{2}\right) = x^2 - \sqrt{2}x + 1$$

and

$$(x - e^{i\frac{3\pi}{4}})(x - e^{-i\frac{3\pi}{4}}) = \left(x - \frac{-\sqrt{2}+i\sqrt{2}}{2}\right)\left(x - \frac{-\sqrt{2}-i\sqrt{2}}{2}\right) = x^2 + \sqrt{2}x + 1$$

Thus the required factorization is:

$$x^4+1 = (x^2 - \sqrt{2}x + 1)(x^2 + \sqrt{2}x + 1)$$

Note Some students may use the following trick: $x^4+1 = (x^4+2x^2+1) - 2x^2 = (x^2+1)^2 - 2x^2 = (x^2+\sqrt{2}x+1)(x^2-\sqrt{2}x+1)$. They should be getting full marks for this of course.