KMB, 14/10/16

M1F Foundations of Analysis, Problem Sheet 2.

1. What are the following sets? Justify your answers.

(a) $\bigcup_{n=0}^{\infty}[n, n+1)$.

(b) $\bigcup_{n=1}^{\infty}[1/n, 1]$.

(c) $\bigcup_{n=1}^{\infty}(-n, n)$.

(d) $\bigcap_{n=1}^{\infty}(-n, n)$.

2. Prove that the set $(0,1)$ (that is $\{x \in \mathbf{R} : 0 < x < 1\}$) has no largest element. (NB: by a "largest element" of a set S I mean an element $x \in S$ such that $\forall y \in S, y \leq x$.)

3.
(a) Prove that if n is an integer and 3 divides n^2 then 3 divides n.
(b) Deduce that $\sqrt{3}$ is irrational.

4. Are the following statements true or false? Proofs or counterexamples required.
(a) If a is irrational and b is irrational then $a+b$ must be irrational.
(b) If a is irrational and b is rational then ab must be irrational.

5. Are the following statements true or false? Proof or counterexample required.
(a) $\forall x \in \mathbf{R}\, \exists y \in \mathbf{R}\, x + y = 2$.
(b) $\exists y \in \mathbf{R}\, \forall x \in \mathbf{R}\, x + y = 2$.

6***.** Prove that $\sqrt{2} + \sqrt{6} < \sqrt{15}$ (NB you may assume the square roots exist).

7. Are the following numbers rational or irrational? Proofs required.
(a) $\sqrt{2} + \sqrt{3/2}$ (hint: if it were rational then its square would also be rational).
(b) $1 + \sqrt{2} + \sqrt{3/2}$.
(c) $2\sqrt{18} - 3\sqrt{8}$.