

M1F Foundations of Analysis, Problem Sheet 1

1. Which of the following statements involving a real number x are true and which are false? You can just write T or F.

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| (a) $x^2 - 3x + 2 = 0 \Rightarrow x = 1.$ | (d) $x^2 - 3x + 2 = 0 \iff x = 1 \text{ or } x = 2.$ |
| (b) $x^2 - 3x + 2 = 0 \Leftarrow x = 1.$ | (e) $x^2 - 3x + 2 = 0 \Rightarrow x = 1 \text{ or } x = 2 \text{ or } x = 3.$ |
| (c) $x^2 - 3x + 2 = 0 \iff x = 1.$ | (f) $x^2 - 3x + 2 = 0 \Leftarrow x = 1 \text{ or } x = 2 \text{ or } x = 3.$ |

2. Suppose P , Q and R are mathematical statements (so they are either true or false). Let's say we know that if Q is true then P is true, and that if Q is false then R is false. Does R imply P ? Write down either a proof, or a counterexample.

3. Say P is true, Q is false, R is false and S is true. Is $(P \Rightarrow Q) \Leftarrow (R \Rightarrow S)$ true or false?

4. Say P , Q and R are true/false mathematical statements, and we know the following:

- (a) $P \Rightarrow (Q \vee R),$
 (b) $\neg Q \Rightarrow (R \vee \neg P)$
 (c) $(Q \wedge R) \Rightarrow \neg P.$

Can we deduce anything about P , Q or R ? For example, is R definitely false? Write down a complete list of possibilities for the truth values of P , Q and R .

5*. Let A be the set $\{1, 2, 3, 4, 5\}$. Which of the following statements are true and which are false? (just write T or F).

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| (a) $1 \in A.$ | (e) $\{1, 2, 1\} \subseteq A.$ |
| (b) $\{1\} \in A.$ | (f) $\{1, 1\} \in A.$ |
| (c) $\{1\} \subseteq A.$ | (g) $A \in A.$ |
| (d) $\{1, 2\} \subseteq A.$ | (h) $A \supseteq A.$ |

6. Now let A be the slightly weirder set $\{1, 2, \{1, 2\}\}$ and let B be the even weirder set $\{1, 2, A\}$. Which of the following statements are true and which are false? (again just write T or F).

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|-----------------------------|---|
| (a) $1 \in A.$ | (e) $1 \in B.$ |
| (b) $\{1\} \in A.$ | (f) $\{1\} \in B.$ |
| (c) $\{1, 2\} \in A.$ | (g) $(\{1, 2\} \in B) \Rightarrow (1 \in A).$ |
| (d) $\{1, 2\} \subseteq A.$ | (h) $(\{1, 2\} \subseteq B) \vee (1 \notin A).$ |

7. Set $A = \{x \in \mathbf{R} : x^2 < 3\}$, $B = \{x \in \mathbf{Z} : x^2 < 3\}$ and $C = \{x \in \mathbf{R} : x^3 < 3\}$. For each statement below, either prove it or disprove it! Be careful with your logic and your exposition.

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|---------------------------------|---|
| (a) $\frac{1}{2} \in A \cap B.$ | (d) $B \subseteq C.$ |
| (b) $\frac{1}{2} \in A \cup B.$ | (e) $C \subseteq A \cup B.$ |
| (c) $A \subseteq C.$ | (f) $(A \cap B) \cup C = (A \cup B) \cap C$ |