KMB, 28/09/01

M1F Foundations of analysis— Problem Sheet 1.

This is assessed coursework. Please hand in solutions to the starred questions (i.e., to questions 1, 3, 4, and 6) at the end of the lecture on Monday 15th October. A dagger indicates a trickier question.

1*) Let A be the set $\{1, 2, 3\}$. Let B be the set $\{1, 2, \{3\}\}$. Let C be the set $\{A\}$. Which of the following statements are true and which are false? (Just write T or F as required)

(a) A has three elements	(f) $A \subseteq B$
(b) C has three elements	(g) $B \subseteq A$
(c) $3 \in A$	(h) $A \in C$
(d) $\{3\} \in A$	(i) $A \subseteq C$
(e) $\{3\} \subseteq A$	(j) $A = C$

2) Write down the mathematical notation for the following two sets (you need not evaluate the sets explicitly):

(a) The set of integers n such that $n^2 > 1000$.

(b) The set of real numbers x such that $x^2 + x + 1 < 0$.

3*) Let A be the set $\{x \in \mathbb{R} : x^2 < 3\}$, let B be the set $\{x \in \mathbb{R} : 0 \leq x \leq 1\}$ and let C be the set $\{x \in \mathbb{Z} : x^2 = 3\}$. Are the following statements true or false? Explain *briefly* your reasoning.

(a) $1 \in A \cap B$.

(b) $1 \in A \cap C$.

(c) $1 \in A \cup C$.

(d) $1 \in A \cap B \cap C$.

4*) Write down proofs of the following statements:

(a) $\sqrt{2} + \sqrt{6} < \sqrt{15}$

(b) The product of a non-zero rational number and an irrational number is always irrational.

5) Disprove the following two statements:

(a) The product of two irrational numbers is always irrational.

(b) The product of two irrational numbers is always rational.

6*) Let x be a real number such that for all positive real numbers y we have $x \leq y$. Prove that $x \leq 0$.

7†) For which integers $n \geq 8$ is $\sqrt{n+8} + \sqrt{n-8}$ rational?