IMPERIAL COLLEGE OF SCIENCE TECHNOLOGY AND MEDICINE

Department of Mathematics

First Year Test Paper - January 2002

M1F - FOUNDATIONS OF ANALYSIS

Date: Wednesday, 9th January 2002

Time: 10.15 - 11.45 a.m.

Credit will be given for all questions attempted, but extra credit will be given for complete or nearly complete answers. The question in Section A will be worth $1\frac{1}{2}$ times as many marks as either question in Section B.
Answer each question in a separate book.
Write your name and the question number prominently on the front of each book.

SECTION A

In each part of this question, just write down your answer. No justification is required.

1. PART I

Say whether the following statements are true or false:

(i) If x and y are irrational, then xy must be irrational too.

(ii) $\sum_{i=0}^{\infty} 10^{-(2^i)}$ is irrational.

(iii) $(\sqrt{3}+i)^{18}$ is a positive real number.

(iv) There exists a map $F : \mathbb{N} \to \mathbb{N}$ that is surjective but not injective.

(v) If I have a box of 20 toys, and at least 12 toys are red, and at least 14 toys are plastic, then at least 6 of the toys must be both red and plastic.

(vi) If $a, b, c, d \in \mathbb{N}$ and $a \equiv b \bmod c$, and $b \equiv c \bmod d$, then $a \equiv c \bmod cd$.

PART II

(i) The number of positive real numbers x such that $x - 2\sqrt{x} = 3$ is

(a) 0, (b) 1, (c) 2, (d) More than 2

(ii) The remainder when 12345671234567 is divided by 11 is

(a) 0, (b) 1, (c) 2, (d) Something else

(iii) The set $\{x \in \mathbb{R} : x^3 - 10x > 1\}$ is

(a) bounded above and below

(b) bounded above but not below

(c) bounded below but not above

(d) not bounded below or above

QUESTION 1 IS CONTINUED OVER

(iv) The block of cheese in my fridge has gone mouldy. I cut off various bits of mould with a knife until the remaining lump of cheese is a convex polyhedron with 13 faces and 22 vertices. How many edges does it have?

(a) 33, (b) 35, (c) 37, (d) impossible to tell

(v) The smallest positive integer that can be written in the form $30\lambda + 52\mu$, where λ, μ are integers, is

(a) 1, (b) 2, (c) 8, (d) some other number.

(vi) The number of injective functions from a set of size 3 to a set of size 2 is:

(a) 0, (b) 3, (c) 6, (d) 9, (e) some other number.

(vii) On 5th December 2001, the number $2^{13,466,917} - 1$ was proved to be prime, and this is currently the largest known prime number. What is its final digit?

(a) 1, (b) 3, (c) 7 (d) 9.

SECTION B

2. (i) Prove that if a is an odd integer, and n is a positive integer, than a^n is an odd integer.

(ii) Deduce that if $n \geqslant 2$ is an integer, then $2^{1/n}$ is irrational

3. (i) Define what it means for a binary relation $\sim$ on a set S to be

(a) reflexive

(b) symmetric

(c) transitive

(d) an equivalence relation

(ii) Define what it means for a collection of subsets $T_1, T_2, \ldots$ of S to be a partition of S.

(iii) If $\sim$ is an equivalence relation on S, define the equivalence class cl (a) of an element $a \in S$, and prove that the equivalence classes of S form a partition of S.

End of paper