

**M1F Foundations of Analysis, Problem Sheet 1**

1. Which of the following statements involving an integer  $x$  are true and which are false? Just write T or F, and perhaps also one remark about why you think this is the answer.

- (a)  $x^2 - 3x + 2 = 0 \Rightarrow x = 1$ .  
 (b)  $x^2 - 3x + 2 = 0 \Leftarrow x = 1$ .  
 (c)  $x^2 - 3x + 2 = 0 \iff x = 1$ .  
 (d)  $x^2 - 3x + 2 = 0 \iff x = 1$  or  $x = 2$ .  
 (e)  $x^2 - 3x + 2 = 0 \Rightarrow x = 1$  or  $x = 2$  or  $x = 3$ .  
 (f)  $x^2 - 3x + 2 = 0 \Leftarrow x = 1$  or  $x = 2$  or  $x = 3$ .

2. Suppose  $P$ ,  $Q$  and  $R$  are mathematical statements (so they are either true or false). Let's say we know that if  $Q$  is true then  $P$  is true, and that if  $Q$  is false then  $R$  is false. Does  $R$  imply  $P$ ? Write down either a proof, or a counterexample.

3. Say  $P$  is true,  $Q$  is false,  $R$  is false and  $S$  is true. Is  $(P \Rightarrow Q) \Leftarrow (R \Rightarrow S)$  true or false?

4. Say  $P$ ,  $Q$  and  $R$  are true/false mathematical statements, and we know the following:

- (a)  $P \Rightarrow (Q \vee R)$ ,  
 (b)  $\neg Q \Rightarrow (R \vee \neg P)$   
 (c)  $(Q \wedge R) \Rightarrow \neg P$ .

Can we deduce anything about  $P$ ,  $Q$  or  $R$ ? For example, is  $R$  definitely false? Write down a complete list of possibilities for the truth values of  $P$ ,  $Q$  and  $R$ .

5\*. Let  $A$  be the set  $\{1, 2, 3, 4, 5\}$ . Which of the following statements are true and which are false? (just write T or F).

- (a)  $1 \in A$ .  
 (b)  $\{1\} \in A$ .  
 (c)  $\{1\} \subseteq A$ .  
 (d)  $\{1, 2\} \subseteq A$ .  
 (e)  $\{1, 2, 1\} \subseteq A$ .  
 (f)  $\{1, 1\} \in A$ .  
 (g)  $A \in A$ .  
 (h)  $A \supseteq A$ .

6. Now let  $A$  be the slightly weirder set  $\{1, 2, \{1, 2\}\}$  and let  $B$  be the even weirder set  $\{1, 2, A\}$ . Which of the following statements are true and which are false? (again just write T or F).

- (a)  $1 \in A$ .  
 (b)  $\{1\} \in A$ .  
 (c)  $\{1, 2\} \in A$ .  
 (d)  $\{1, 2\} \subseteq A$ .  
 (e)  $1 \in B$ .  
 (f)  $\{1\} \in B$ .  
 (g)  $(\{1, 2\} \in B) \Rightarrow (1 \in A)$ .  
 (h)  $(\{1, 2\} \subseteq B) \vee (1 \notin A)$ .

7. Set  $A = \{x \in \mathbf{R} : x^2 < 3\}$ ,  $B = \{x \in \mathbf{Z} : x^2 < 3\}$  and  $C = \{x \in \mathbf{R} : x^3 < 3\}$ . For each statement below, either prove it or disprove it! Be careful with your logic and your exposition.

- (a)  $\frac{1}{2} \in A \cap B$ .  
 (b)  $\frac{1}{2} \in A \cup B$ .  
 (c)  $A \subseteq C$ .  
 (d)  $B \subseteq C$ .  
 (e)  $C \subseteq A \cup B$ .  
 (f)  $(A \cap B) \cup C = (A \cup B) \cap C$