Imperial College London
M2PM2 Algebra II, Progress Test 1, 26/10/2012.

Name (IN CAPITALS!):...**CID**: ....

Q1. [5 marks]

True or false? Just write the answers in this question – no working needed. **A right answer gains you $\frac{1}{2}$ of a mark – but a wrong answer *loses* you $\frac{1}{2}$ of a mark**, so if you're really not sure then you might want to leave the question out (and get 0 marks).

i. Every group of order 6 has a subgroup of order 4.
ii. If G and H are two groups of order 8 then G is isomorphic to H.
iii. If G is a group and $g \in G$ is an element of order 4, then G must be finite.
iv. If G and H are groups, and $f : G \to H$ satisfies $f(xy) = f(x)f(y)$ for all $x, y \in G$, then G is isomorphic to H.
v. If σ is a 2012-cycle in a symmetric group, then $\mathrm{sgn}(\sigma) = -1$.
vi. If G, H and K are groups, and $G \cong H$ and $H \cong K$, then $K \cong G$.
vii. Every symmetric group S_n, $n \geq 1$, contains an element of signature -1.
viii. If $g \in S_n$ then $\mathrm{sgn}(g^3) = \mathrm{sgn}(g)$.
ix. There exists a symmetric group that contains at least a million elements with signature -1.
x. There are subgroups X of S_{10} and Y of S_{11} such that $X \cong Y$.

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Q2. [10 marks]

This question is about the dihedral group D_{10}. You may assume any results from the course (but it would be great if you stated them clearly). Let's use the standard notation from the course, so D_{10} contains ρ (a rotation by the angle $2\pi/5$), and a reflection σ.

i. Say $H \subseteq D_{10}$ is a subgroup, and assume $H \neq D_{10}$. Prove that H is cyclic. Hint: consider the possibilities for the order of H.

ii. List all the subgroups of D_{10}. How many are there? How many are there up to isomorphism?

iii. On the first example sheet you proved that $\sigma\rho = \rho^{-1}\sigma$, and you can *assume* that here. The element $\rho\sigma\rho^2\sigma\rho^3\sigma\rho^4\sigma$ is a power of ρ – which one?

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Q3. [10 marks]

In this question, you can again assume any results from the course without proof. Notation: S_n denotes the usual symmetric group.

i. Proof or counterexample: if $g, h \in S_n$, then does $\mathrm{sgn}(gh) = \mathrm{sgn}(hg)$?
ii. List all the possible cycle-shapes for the elements of S_4. For every one of these cycle-shapes, say whether an element with this cycle shape is even or odd.
iii. Prove that if $g \in S_4$ and $\mathrm{sgn}(g) = +1$ then g can be written as the product of two 2-cycles.
iv. Is is true that every $g \in S_5$ with $\mathrm{sgn}(g) = +1$ can be written as a product of two 2-cycles? Proof or counterexample required.