**Name (In CAPITAL letters)**..........................................................................................

**CID**: ................................**Course: M2PM2 Algebra II, Progress Test 1, 31/10/2014.**

**This test has two questions, Q1 and Q2. They are worth the same number of marks.**

**Q1.** Let $G$ be the group $D_{10}$, and let $\rho$ and $\sigma$ be as in lectures (so $\rho$ is a rotation by $2\pi/5$, with order 5, and $\sigma$ is a reflection, with order 2). You may *assume* that $D_{10} = \{e, \rho, \rho^2, \rho^3, \rho^4, \sigma, \rho\sigma, \rho^2\sigma, \rho^3\sigma, \rho^4\sigma\}$ (and that these ten elements are distinct!), that the reflections in $D_{10}$ are $\rho^i\sigma$ for $0 \leq i \leq 4$, and you may also assume that $\sigma\rho = \rho^{-1}\sigma$ when doing this question.

**i.** Proof or counterexample: $\rho$ and $\rho^{-1}$ are the only elements of order 5 in $D_{10}$.
**ii.** The element $\rho\sigma\rho^2$ is a reflection; write it (with proof) as $\rho^m\sigma$ for some appropriate $m$.
**iii.** How many cyclic subgroups does $D_{10}$ have?
**iv.** Let $H$ be a subgroup of $D_{10}$ that contains at least two reflections. Prove that $H = D_{10}$.

**Name (In CAPITAL letters)**..........................................................................................

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**Q2.** In this question you can assume any results you need from the course, unless you are explicitly asked to prove them.

**i.** Prove that if $G \cong H$ and $H \cong K$ then $G \cong K$.
**ii.** Is $A_4$ isomorphic to $D_{12}$?
**iii.** Let $G$ be the group $S_3$ and define $f : G \to G$ by $f(g) = g^{-1}$. Is $f$ a bijection? Is $f$ an isomorphism? Justify your answers.
**iv.** What is the signature of $(1\ 2\ 3)(4\ 5)(6\ 7\ 8) \in S_9$? Explain your answer, but you can quote any result from the course without proof.
**v.** Write down an element of $S_9$ with such that its order is even and its signature is $+1$.