Name (In CAPITAL letters)..

CID: ....**Course: M2PM2 Algebra II, Progress Test 1, 15/10/2013.**

This test has two questions, Q1 and Q2. They are worth the same number of marks.

Q1. For each assertion below, either prove it (if it is true) or give a counterexample (if it is false). Note that you may of course assume, without proof, any standard results from the first year.

i. Proof or counterexample: Every group of order 12 is cyclic.
ii. Proof or counterexample: every group of order 7 has a subgroup of order 6.
iii. Proof or counterexample: there exists a group G and an element $g \in G$ such that $g^{1000} = e$ (e the identity) but $g^{1001} \neq e$.
iv. Proof or counterexample: If the order of every element of a group G is 1 or a prime number, then G must be cyclic.
v. Proof or counterexample: If G is a group and $a, b \in G$, and if $G = \langle a \rangle = \langle b \rangle$, then $a = b$.

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Q2. In this question, you can just write down the answer for part i, but for all the other parts don't just write the answer – also justify your answer (i.e., give details). In this question, e denotes the identity element of a group. You may use any standard facts about dihedral groups in this question.

i. How many rotations are there in D_{12}? How many reflections?
ii. How many subgroups of order 2 does D_{12} have?
iii. Using any standard facts about dihedral groups, prove that if σ is a reflection and ρ is the rotation by $2\pi/6$ (i.e., the usual notation) then $\sigma\rho^n\sigma = \rho^{-n}$ for any integer $n \geq 1$.
iv. Is there an element z in D_{12} such that $z \neq e$, but $zg = gz$ for all $g \in D_{12}$?