M2PM2

Plan:

1. More groups
2. More vector spaces

§1: GROUPS

(again)

Vague reminder:

a **group** is

* a set G
* a way of "multiplying 2 elements together"
* some axioms.

Examples

① $G = \mathbb{Z}$ or $\mathbb{Q}$, or $\mathbb{R}$ or $\mathbb{C}$

Group law : $+$

② Matrix groups like $GL_3(\mathbb{R})$

= 3×3 invertible matrices

Group law : $\times$

③ Symmetric groups

- start with $n \in \mathbb{Z}_{\geq 1}$

Symmetric group S_n

= permutations of set $\{1, 2, 3, \ldots, n\}$

Group law: " $\circ$ "

composition of functions

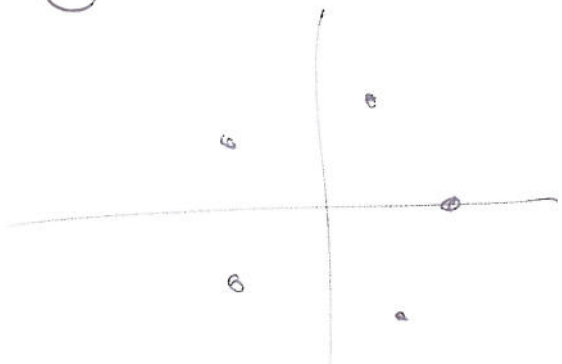
4 $n \in \mathbb{Z}_{\geq 1}$

Cyclic group of order n called C_n

$$C_n = \{ z \in \mathbb{C} : z^n = 1 \}$$

$$= \{ 1, \omega, \omega^2, \dots \omega^{n-1} \}$$

$$\omega = e^{2\pi i/n}$$

C_5 : 

Group law : $\times$

5 Stupid example!

$G = \{ \text{my cat} \}$

(my cat) $\times$ (my cat) = my cat.

That is a group.

General theory that you already did:

1. Subgroups

G: a group

$H \subseteq G$: a subset

H is a subgroup if

(a) $e \in H$ (e = identity element of G)

(b) If $x, y \in H$ then $x.y \in H$ ($\cdot$ = group law in G)

(c) if $x \in H$ then $x^{-1} \in H$ (x^{-1} means inverse of x)

The point: H is then also a group.

2. (special case)

Cyclic subgroups

G: a group

$a \in G$: any old element.

Usually $\{a\} \subseteq G$ is not a subgroup. But

$$\langle a \rangle := \{a^n : n \in \mathbb{Z}\}$$

is a subgroup.

$\langle a \rangle$ is the cyclic subgroup of G generated by a.

BASIC FACTS ABOUT $\langle a \rangle$:

$\langle a \rangle$ is a set & it's often finite. If it's finite, then its size, $|\langle a \rangle|$, is the order of a.

If $\langle a \rangle$ is an infinite set, we say a has infinite order.

Notation: $o(a)$ = order of a.

IMPORTANT THINGS.

Say $o(a) = k$, finite

* $a^k = e$, & k is the smallest positive integer with this property.
* $a^m = a^n$ if & only if $m-n$ = multiple of k.

Exercise

Find a regular 7-sided figure (eg 20p)

Turn it around.

That's a cyclic group of order 7.

It's $\langle a \rangle$, where

$a =$ "turn the 20p around by an angle of $\frac{360}{7}$"

Turn a 20p round by $\frac{360}{7}$ degrees, 9 times.

Same as turning it twice.

$9 - 2 =$ multiple of 7.