Cassels–Froehlich errata.

July 18, 2013

<div align="right">

Consider what God has done:
Who can straighten what he has made crooked?

Ecclesiastes 7:13

</div>

1 These errata.

When J-P. Serre heard that the London Mathematical Society was going to reprint the book known to us all as "Cassels-Froehlich", he urged them to make a correction to his Ugly Lemma (a correction already indicated in his collected works). I was independently motivated to try and compile a list of errata for the book (despite having read the claims in the original preface of the source of the inaccuracies!), because I had once spent two hours stuck with a non-commutative commutative diagram, the error in which I finally traced back to a minor slip in the first printing of the book. It occurred to me that several of my colleagues the world over may well know of other errata, and that perhaps one could use the internet to try and gather slips known to various other people. I posted a request for errata on the number theory mailing list `NMBRTHRY@listserv.nodak.edu`, at the internet newsgroup `sci.math.research`, at the website `mathoverflow.net`, and I also individually approached (via email) several individuals whom I knew to be careful readers. Ultimately I would like to thank R. Bellovin, B. Conrad, K. Conrad, F. Herzig, H. Lenstra, J. Oesterlé, R. Schoof, and J-P. Serre, each of whom provided a list which was either substantial or contained at least one gem, and I would also like to thank the many other people who provided me with smaller lists. All are documented below, and, as will be immediately apparent to anyone who takes a careful look at them, almost all of them are utterly trivial. I would be happy to hear from anyone who spots any errata not on this list. Any errors in these errata are of course entirely my fault; I will keep an updated list on my web site.

As far as I know, the hardback version of the book (published in 1967 and again in 1969) and the paperback versions (1986, 1987, 1990) are the same (that is, no corrections to the text were made in the mean time). The errata below hence apply to both editions. Notation: "p. x l. y" means page x, line y from the top (not counting running headers) and "p. x l. -y" means y lines up from the bottom of page x (counting lines in footnotes, and counting equations as one line).

Kevin Buzzard

1.1 Version history.

This is the version of 18th July 2013. A full version history:

16 Feb 2010: original list.
01 Mar 2012: added a correction (p. 130 l. -14).
13 May 2012: added a correction (p. 18 l. 5).
18 Jul 2013: added some corrections (p. 49 l. -17, p. 55 l. 4, p. 69 l. 1, p. 163 l. -5)

2 Corrections.

This section consists of mathematical slips that may confuse or mislead the reader (if they are learning the theory from the book).

p. 3 Proposition 1: The proof of the proposition is in Chap. II §5, not §10, and furthermore what is proved there is the uniqueness of the completion, rather than the uniqueness of the extension (which is also true, but is never used in the book).

p. 18 l. 5: $t_{L/K}$ should be $t_{L/F}$.

p. 29 l. 12: $1 + \Pi^n t_{L/K}(x)$ should be $1 - \Pi^n t_{L/K}(x)$.

p. 45 l. 5: The right hand side of the equation should be $n + 1$ not n.

p. 49 l. -17: "choice of α" should be "choice of π".

p. 52 l. -15: K should be V.

p. 52 l. -12: the subscripts should be outside the norm signs in the second equation.

p. 53 two lines above section 9: the last subscript n should be an N in the first norm on the right hand side of the equation.

p. 55 l. 4: $h(\beta)$ should be $h(\beta_j)$ (and nowadays this would be written $h(\overline{\beta}) \mapsto h(\beta_j)$); moreover, AX should be $A[X]$.

p. 56 first line of Theorem: second k should be $|\,|$.

p. 69 l. -15: it should say "(i) S contains all the v with $|\alpha_v|_v \neq 1$" (note that the footnote can be used to check that this set is finite).

p. 69 l. -14: $|\xi_v|_v < \frac{1}{2}C$ should be $|\xi_v|_v < \frac{1}{2C}$.

p. 73: interchange $\mathrm{con}_{L/K}$ and $\mathrm{con}_{L/k}$ in (19.3).

p. 75 l. 1: (19.9) should be (19.10).

p. 78 l. 6: b_{ij} should be b_{1j}.

p. 78 equation (A.24): the second $N_{V/P}$ should be $N_{V/R}$, and similarly two lines later.

p. 79 l. -8: change "$\beta_1, \beta_2, \ldots, \beta_m$" to "$\beta_1 \beta_2 \ldots \beta_m$".

p. 80 l. 13: replace "into k" by "into M".

p. 83 equation (C.3): $|f(\alpha)|/|f'(\alpha)|$ should be $|f(\alpha_0)|/|f'(\alpha_0)|$.

p. 88 l. -11: $(1 - \zeta^i)$ should be $(1 - \zeta)^i$.

p. 88 l. -10: change "$i = 1, \ldots$" to "$i = 0, 1, \ldots$"

p. 88 Lemma 6: the discriminant of a number field is typically viewed in the book as an ideal, rather than an integer, so statements about discriminants of cyclotomic fields here should be viewed "up to sign".

p. 90 Lemma 2: we should assume that $[L : K] = n$, and the last clause in the statement of the lemma should say "(b must have order n in $K^{\times}/(K^{\times})^n$)".

p. 90 Lemma 3: the cyclic extensions should be assumed to have degree n.

p. 91 l. 7: the ν should be above the arrow in $L^{\times} \to L^{\times}$.

p. 91 l. -3: "Chapter, II Section 16" should be "Chapter II, Section 10".

p. 92 l. -6: replace $\mathfrak{o}$ by $\mathfrak{o}_{\mathfrak{p}}$.

p. 92 l. -2: f_j^* should be g_j^*.

p. 95 l. 1: The (unspecified) G-action on $\mathrm{Hom}(\Lambda, X)$ is given by $(g.f)(\lambda) = f(\lambda g)$.

p. 98: Replace the second sentence of section 4 by "If A' is a G'-module, we can form the G-module $A = \mathrm{Hom}_{G'}(\Lambda, A')$. We give A the following G-module structure: if $\phi \in A$ then $g.\phi$ is the homomorphism $h \mapsto \phi(hg)$." (the given "left action" is not a left action).

p. 99 l. -17: should say $\mathrm{Inf} : H^q(G/H, A^H) \to H^q(G/A)$.

p. 99 Equation (4.2) should be $H^q(G, A^t) \to H^q(G, A)$.

p. 101 l. 13 the Hom should be $\mathrm{Hom}(\mathbf{Z}[G/H], A^*)$.

p. 104 both horizontal maps in the displayed commutative diagram should be labelled $\hat{\delta}$.

p. 106 The left hand side of of equation (7.3) should be $d(f.g)$

p. 118 Corollary 2, and p. 119 Corollary 3: the limits here are unwritten but should be over U.

p. 124 displayed argument at bottom of page: $f(y)^{-1}$ would be a clearer thing to say than $f^{-1}(y)$ (three times).

p. 126 l-13: Proposition 4 is correct, but the given proof is not. The condition $H^1(H, A) = 0$ is equivalent to $H^1(H/(H \cap U_i), A^{H \cap U_i}) = 0$ for all open normal U_i of G, but one cannot replace $A^{H \cap U_i}$ by A^{U_i}. The most natural proof of Proposition 4 is simply to use the Hochschild-Serre spectral sequence.

p. 130 l. 1, replace §2.7 by §2.8.

p. 130 l. 14, replace §2.5 by §1.5.

p. 130 l. -14, delete "(that is, $\mathbf{Z}$-injective)".

p. 131 last line before Corollary 1, replace §2.7 by §2.8.

p. 131 Corollary 2 should start "Let L/K be a Galois extension. . .".

p. 131 l. -10: $H^2(\hat{\mathbf{Z}}, K_{nr})$ should be $H^2(\hat{\mathbf{Z}}, K_{nr}^{\times})$.

p. 133 Proof of Corollary 1, first sentence should be "Note that if L/K is Galois then a less. . .".

p. 135, part b) of Lemma 4. Replace $H^q(H, M)$ by $H^q(K/H, M^H)$ and replace $\hat{H}^0(H, M)$ by $\hat{H}^0(K/H, M^H)$.

p. 135 l. -6. Replace "to G/H" by "to H".

p. 135 l. -4: insert "the order of" after "hypothesis".

p. 138 Application: one should appeal to Cor. 1 rather than Cor. 2.

p. 140 l. 3 of section 2.3: replace $H^2(G, \mathbf{Z})$ by $H^1(G, \mathbf{Q}/\mathbf{Z})$.

p. 140 l. -8: replace s by s_α.

p. 140 l. -2: replace "Prop. 2" by "Prop. 1". Also, it might be clearer to define s_α' to be $(\alpha, L'/K)$, and then note that $s_\alpha \mapsto s_\alpha'$ follows from the equality of character values (rather than equality of the character values following from Prop. 1).

p. 141 first line after the first displayed diagram: replace K'^{ab}/K by K'^{ab}/K'.

p. 141: in the second displayed diagram, replace θ_K' by $\theta_{K'}$.

p. 144 l. -4: change "generated" to "topologically generated".

p. 145: $G = G_{L/K}$.

p. 145: In the statement of Proposition 6, replace "of Proposition 3" by "of Proposition 5".

p. 148 l. 7: $(X) = \ldots$ should be $f(X) = \ldots$.

p. 150 l. -17: left hand side should be $f \circ \phi^{(p)} - \phi^{(p)} \circ g$.

p. 151 l. 13: the last $[a]$ should be $[b]$.

p. 152 l. -16: $G(K^n/K)$ should be $G(K_\pi^n/K)$.

p. 154 l. 7: $\hat{A}[[X]]$ should be $\hat{A}_{nr}[[X]]$.

p. 154 l. 17: replace the first (or both) r_π by $r_\pi(\omega)$.

p. 154 l. 19: strictly speaking one now has to check that the K_π from section 3.6 is the same as the K_π from section 2.8 (but this follows relatively easily from what we have shown already).

p. 154 l. 2 of section 3.8: replace "2.3" by "2.7".

p. 154 l. -8: replace I_K by I_K'.

p. 156 l. 3: replace "3.3" by "3.4".

p. 156 l. 10: replace β_j by β^j.

p. 157 l. 1 of section 4.2: L/K should be assumed abelian.

p. 157 l. -2: (χ) should be $f(\chi)$.

p. 162 footnote: the Artin–Tate notes "Class Field Theory" are available from AMS–Chelsea (2008).

p. 164 l. -7: the right hand side should be $\tau \sigma_w \tau^{-1}$.

p. 170 l. -18: replace "make U^S an arbitrarily small. . ." by "ensure U^S is contained in an arbitrarily small. . ."

p. 175 displayed commutative diagram: the left hand objects should be $\mathcal{M}^* = M_w^*$ and K_v^*.

p. 175, two lines after the commutative diagram: replace $N_{M/K}$ by $N_{M/K}(J_M)$.

p. 177 l. -14: replace §1.4 by §1.2.

p. 179 l. -19: replace $w \in S_L$ by $w \in T$.

p. 180 l. 9: S should be assumed to contain all archimedean and ramified primes.

p. 183 l. 1: change "Proposition 2" to "Proposition 2.3".

p. 183 equation (7): replace the second K by K^*.

p. 183 l. -7: replace "(5)" by "(6)".

p. 185 l. -11: "splits over K" should be "splits over L".

p. 187: in Corollary 9.8, L/K should be assumed finite.

p. 189 l. -2: replace "§2.4" by "§2.3".

p. 190: in the second displayed commutative diagram (line 8), the source and target of the map N going down on the left should be K_V^* and $\mathbf{Q}_p^*$.

p. 190 l. 10: replace 3.1 by 3.2.

p. 190 l. 11: replace $a \in K$ by $a \in K^*$.

p. 190 l. 12: replace $\mathbf{Q}$ by $\mathbf{Q}^*$ and K by K^*.

p. 190 l. -12: the source of ψ_p should be $\mathbf{Q}_p^*$.

p. 190 l. -6: replace $\alpha \in \mathbf{Q}_p$ with $\alpha \in \mathbf{Q}_p^*$.

p. 190 l. -6: replace $\psi_p(\alpha)|\mathbf{Q}_p$ with $\psi_p(\alpha)|\mathbf{Q}_p^{nr}$.

p. 191 l. -9: replace $p = q =$ by $p = q = l$.

p. 192 ll. 9–10: K is not assumed Galois over $\mathbf{Q}$ so one should multiply m by $\mathrm{lcm}(1, 2, \ldots, [K : \mathbf{Q}])$ (the local degrees may not divide $[K : \mathbf{Q}]$).

p. 193 l. -2: replace "algebraic closure" by "separable closure".

p. 195 l. 14: $\beta_2(\mathrm{infl}\, b)$ should be $\beta_2(\varepsilon_2(\mathrm{infl}\, b))$.

p. 196 l. 5: the equation should be $\mathrm{Im}(\beta_1) \supset \mathrm{Im}(\mathrm{inv}_1)$.

p. 201 l. -6: the finite set S should be assumed large enough that conditions (i)–(iii) of p. 182(2) hold (so we can apply consequence 9.5 later).

p. 201 l. -5: K^{*n} should be K_v^{*n}.

p. 202 l. -8: replace H by H' and replace second $(C_{K'} : H')$ by $(C_{K'} : H_1')$.

p. 207: 2–3 lines below displayed formula for $\psi_{\mathfrak{p}}(x)$, replace "determined by" by "trivial on".

p. 209 ll. 11–19: most of the statements about indexes seem to be wrong or at least ambiguous, but the conclusion (the final formula) is correct.

p. 211 l. 14: insert "primitive" before "Dirichlet".

p. 211 l. -13: replace "0 or 1" by "1 or 0".

p. 212 l. 14: $+c_{\mathfrak{p}}$ should be $-c_{\mathfrak{p}}$.

p. 214 l. 15: the definition of c should be $c = [k : \mathbf{Q}]$, and furthermore $g(s)$ should not be taken to be the function given, but rather the function $-L'(s, \chi)/L(s, \chi) - g(s, \chi)$, with $g(s, \chi)$ defined on p. 213.

p. 221 l. -4: replace $\mathfrak{P}$ with $\mathfrak{q}$.

p. 222 l. -9: left hand side should be $\chi^*(\mu_0^m)$.

p. 225 l. 11: after "can be taken to be rational integers" add "(provided one does not insist that the extensions K/Ω_l be cyclic)".

p. 236 l. 5: replace "2.5" by "1.2, Prop. 1".

p. 242: in the second big displayed diagram, the E_K in the top left should be an E_k.

p. 322 l. 2: the exponent of $\mathfrak{p}$ should be $(d + n)s$ not $(d + n)^s$.

p. 353: equation $(\ast\ast)$ should read $\left(\frac{\lambda}{b}\right) = \prod_{v \in S}(b, \lambda)_v$.

p. 357 l. -14: the left hand side of the equation should be $f(tX + Y)$.

p. 360 last line of exercise 5.1: replace "4.3" by "4.4".

p. 366: under "Tchebotarev, N.", also list pages 165 and 227.

3 Typesetting issues and typos.

These are just slips which are unlikely to derail the reader.

p. 26 l. -4: the first *Hom* should not be in italics.

p. 30 l. -16: "the L_1" should be "that L_1"

p. 33 l. 18: change $0(h^2)$ to $O(h^2)$.

p. 33 ll. -11 and -14: change Z to $\mathbf{Z}$.

p. 45 l. -8: change $b \in \mathbf{Z}$ to $b \in \mathbf{Z}_{\geq 1}$.

p. 50 l. -12: add a ")" at the end.

p. 52 l. 12: $(-)^{n-1}$ is now usually written $(-1)^{n-1}$.

p. 52 l. -1: the absolute values signs are misprinted.

p. 53 l. 2: the absolute value sign on the right is misprinted.

p. 54 l. -5: italicize A (twice).

p. 55 l. 7: change $\oplus$ to $\bigoplus$.

p. 57 l. -11: change $\oplus$ to $\bigoplus$.

p. 60 l. 2: change $\oplus$ to $\bigoplus$.

p. 61 l. 8: the subscript in β_p is too low.

p. 62 l. 1: delete "$\alpha \in$ R.H.S. and that"

p. 69 l. 1: "$\{\alpha_v\} \subset J_k$" should be "$\{\alpha_v\} \in J_k$" in the definition.

p. 73 l. 6: replace "vica" by "vice".

p. 74 equation (19.14): change $\oplus$ to $\bigoplus$ (twice).

p. 76 l. -6: $(-)^n$ is now usually written $(-1)^n$.

p. 78 l. 4: do not italicize *End.* (twice)

p. 84 l. -1: italicize X and put a comma after α.

p. 85 l. -4: Q_p should be $\mathbf{Q}_p$.

p. 86 l. 7: Q should be $\mathbf{Q}$.

p. 86 l. 16: "numb$f$ering" $\rightarrow$ "numbering", and the f should be moved one line lower onto line 17.

p. 86 second line of statement of Corollary 2: Q should be $\mathbf{Q}$.

p. 87 l. 5: Q should be $\mathbf{Q}$.

p. 87 l. -6: Q_p should be $\mathbf{Q}_p$ twice.

p. 87 ll. -4 -1: Q_p should be $\mathbf{Q}_p$.

p. 88 l. 17: $Z[\zeta]$ should be $\mathbf{Z}[\zeta]$.

p. 89 l. -5: an "automorph" is usually called an "automorphism" nowadays.

p. 91 l. 9: the arrow below ν is missing.

p. 106 l. 14: $f^o g$ should be $f.g$.

p. 115 l. -3: "follows from (iii)" should be "follows from (ii)".

p. 118 l. -15: change $g \rightarrow (gH_i)$ to $g \mapsto (gH_i)$ (see p. xviii)

p. 119 l. 16: change $g \rightarrow (gH_i)$ to $g \mapsto (gH_i)$ (see p. xviii)

p. 123 l. -19: C should be C.

p. 123 ll. -8,-9,-10: change , .., to , ...,

p. 129 l. 10: insert another) after $[k : \mathbf{F}_p]$.

p. 133 l. 2 of Corollary 1: do not italicize *Br*.

p. 135, l. 6 of lemma 4: replace the second "M)" by "M))".

p. 139 l. 14: put ")" before the final ".".

p. 141 l. -15: the q here is (clearly) not the same q as on line 9.

p. 141 l. -8: Z should be $\mathbf{Z}$.

p. 143 ll. -3 to -2: change "Lubin Tate's" to "Lubin-Tate" (the theory is now commonly called Lubin-Tate theory).

p. 145 l. -1: the subscript in $\mathbf{R}_+^\times$ is cut off a bit.

p. 147 l. 20: insert ")" before ".".

p. 147 l. -17: now K is a local field.

p. 150 proof of Proposition 1: strictly speaking (c) is not proved that way (but it follows from (a), (b), (d)).

p. 150 l. -10: replace "left-and" by "left- and"

p. 155 l. 18: target of i_G should be $\mathbf{Z} \cup \{\infty\}$.

p. 155 l. -11: change G^v to G^v.

p. 157 l. 9: replace "intertia" by "inertia".

p. 158 l. -4: replace "$K'|K$" by "K'/K".

p. 163 l. -5: The middle term in the displayed equation should be $[G(L_w/K_v)]$ rather than $G(L_w/K_v)$.

p. 168 l. 5: replace F by F.

p. 168 l. -16: replace C by C.

p. 173: Title of section 5.6 should be "Number Field Case".

p. 174 l. -8: The notation G^v is used here for the local Galois group, whereas subscripts have been used earlier in the article. By the next page subscripts are being used again (see p. 175 l. -8).

p. 177: Last symbol on page should be J_L.

p. 179 l. 12: insert ")" before the second "=".

p. 182 l. 14: $K^* \cap J_{K,S}$ is better than $K \cap J_{K,S}$.

p. 182 l. -5: remove the ")" after "defined"—the bracket opened on l. -6 will be closed on l. 2 of p. 183.

p. 183 l. 5: perhaps $(K_v^*)^n$ is better than K_v^n.

p. 185 l. -18: local Galois groups are now again, temporarily, being denoted G^v rather than G_v. They are still superscripts on p. 188 l. -4 but are back to subscripts by p. 189.

p. 188 l. 5: the last "a" should be italicised.

p. 189 l. -14: replace "$\theta(a) = 0$" by "$\theta(a) = 1$".

p. 190 l. 9: replace "Chaper VI" by "Chapter VI", and perhaps a better reference than §2.1 would be §2.4.

p. 192 l. -11: replace "infiinte" by "infinite".

p. 197 l. 6 of "case $r = -2$": H^{-2} should be $\hat{H}^{-2}$, and two lines later, H^{-1} should be $\hat{H}^{-1}$.

p. 198 l. -10: a reference for this "fundamental duality theorem" is Theorem 6.6 in Chapter XII of Cartan-Eilenberg.

p. 201 l. -17: ψ_k should be ψ_K.

p. 211: the definition of $\Phi(s, \chi)$ is out by a non-zero constant from the now standard definition of the completed L-function attached to χ (but of course this does not affect the functional equation).

p. 211 l. -7: replace "seq" by "$seq.$"

p. 214 l. 13: if $\eta = 0$ then this equation should be interpreted as saying that the sum is $o(x)$.

p. 215 l. 6: the right side is missing an l.

p. 215 l. -1: insert "relative to k" after "absolute first degree".

p. 224 l. -4: this is not really "another way of arriving at a contradiction" because this sort of argument is precisely how one proves linear independence of characters.

p. 225 ll. 1–3: the proposed procedure seems to involve inverting a submatrix that depends on the choice of p (but the Brauer argument mentioned later on makes this remark moot).

p. 229 l. 9: the f on the left side is cut off a bit.

p. 230 l. 7: The author is using the Prime Ideal Theorem in k as well (and could use the Chebotarev density theorem instead).

p. 240 ll. -2 and -4: change $\to$ to $\mapsto$.

pp. 248–249: change "$math.$" to "$Math.$" in the references.

p. 249 l. 5: "$Nauk.$" should be "$Nauk$".

p. 252 l. 1: change "sem -simple" to "semi-simple".

p. 284 ll. -8, -11, -12: change $\mathbf{Q}(i)$ to $\mathbf{Z}[i]$.

p. 290 l. 8: change "Wood's Hole" to "Woods Hole".

p. 292 l. -16: insert colon after "field".

p. 293 l. 17: change "inversible" to "invertible".

p. 298: in Lemma 2, the field diagram is hard to read in parts. The bottom line is from k to k' and is labelled with the letter E.

p. 303 l. 14: remove comma after "enough".

p. 303 l. 25: remove comma after "difficult".

p. 303 ll. -2,-3: the Brumer reference appeared in Michigan Math. J. **13** (1966), 33–40.

p. 304 ll. 9–10: the abbreviation for the journal should match Shafarevich's second paper on p. 249.

p. 312 l. -7: $L(k^+ - 0)$ should be $L(k^+ - \{0\})$.

p. 322 l. 8: in the integral, the bold d is slightly too high.

p. 333 l. 14: change $|\ \widehat{f}(\mathfrak{a}\xi)|$ to $|\widehat{f}(\mathfrak{a}\xi)|$.

p. 343 l. 18: Probably better notation would be to choose a solution $t_{\mathfrak{p},0}$ (not $t_{\mathfrak{p}}$) and then set $t_{\mathfrak{p}} = t_{\mathfrak{p},0} + t$.