Algebraic number theory

Test 2

March 12, 2012

You can use any results from lectures without proof.

1. List all quadratic fields of discriminant D such that $|D| \leq 12$. (No proof is required.)

In the next two questions $K = \mathbb{Q}(\sqrt{-13})$.

2. Let $P \subset \mathcal{O}_K$ be a prime ideal such that $2 \in P$.
   (a) Find two elements α, $\beta \in \mathcal{O}_K$ such that $P = (\alpha, \beta)$.
   (b) Prove that the class of P in the class group $\mathrm{Cl}(K)$ has order 2.
   (c) Deduce that $\mathcal{O}_K$ is not a Euclidean domain.

3. Factorise $(1 + \sqrt{-13})$ into prime ideals in $\mathcal{O}_K$.