Yuri Manin

Alexei Skorobogatov

March 10, 2023

Yuri Ivanovich Manin, the only child of Ivan Gavrilovich Manin and Revekka Zinovievna Miller, was born on the 16 February 1937 in Simferopol (Crimea). His father, a lecturer in geography, was a son of an illiterate Russian peasant. His mother, a postgraduate student of Russian literature, was a daughter of a playwright and journalist of Jewish origin born in Hughesovka (modern Donetsk in the Ukraine).

It seems that until very recently, Yuri Manin rarely talked about his childhood. I asked him several years ago if he ever wanted to write an autobiography. He said that he tried but eventually had to give up because writing brought him too much pain.

Yuri Manin's early personal history is an intense family tragedy caused by the catastrophe of the Second World War[1]. In the fifth year of his life, in the face of the Wehrmacht's rapid advance, the Manin family together with the grandparents Zinovy and Hannah Miller fled to the North Caucasus and then further still, across the Caspian Sea, to Chardzhou in the Soviet Central Asia (modern Turkmenistan). Refugees in their own country, they had no place to stay and no means of existence. During the year of 1942, Yuri's grandmother fell ill and died, soon thereafter his grandfather committed suicide. Later in the year Yuri's father volunteered to fight in the Red Army and perished in the war. After the liberation of Crimea, Yuri and his mother returned to Simferopol only to find the apartment where Yuri lived before the war occupied by another family. Yuri and Revekka became effectively homeless, though they were allowed to move in to Yuri's aunt's communal apartment. The difficult post-war childhood was further complicated when Yuri's mother lost her job in the Soviet state antisemitic campaign of 1948.

While at school, Yuri reads I.M. Vinogradov's book 'Elements of number theory' and sends the author a letter with a generalisation of the formula for the number of integer points in a circle.

Yuri's life radically changes when he becomes an undergraduate in the Department of Mechanics and Mathematics of the Moscow State University in 1953, the year of Stalin's death and the inauguration of the grandiose Main Building of the

[1]An account of the beginning of the family's evacuation was written by Yuri Manin's grandfather Zinovy Miller. The Russian original with Yuri Manin's introduction is available electronically [9].

University[2]. Igor Shafarevich approaches Yuri and offers to become his supervisor, and – already in 1956 – Yuri's first published paper 'On cubic congruences to a prime modulus' appears in *Izvestia*, a leading Soviet mathematical journal. The paper gives an elementary proof of Hasse's bound for the number of points on an elliptic curve over the finite field with p elements. Manin's precocious talent manifests itself in a series of papers on algebraic curves, many of which are nothing short of miraculous. The high point of this period is Manin's proof of the functional analogue of Mordell's conjecture in characteristic zero (1963). This ingenious proof is based on the discovery of a major new method involving the algebraisation of the theory of Picard–Fuchs differential equations and what Grothendieck later calls the Gauss–Manin connection. It has become a standard tool when dealing with cohomology of families of varieties over a base. In the words of Robert Coleman, "this work is testimony to the power and depth of Manin's intuition" [1]. Another foundational result of this period is Manin's classification of commutative formal groups over fields of finite characteristic. For these achievements he was awarded the Lenin prize in 1967, one of the most prestigious awards of the Soviet Union usually given to composers, ballet dancers and rocket scientists.

The decade from the mid-1950s to the mid-1960s was an amazing time in Russia. The liberalisation of society brought forth the coming of age of an optimistic new generation. Physics and mathematics, largely spared from the Stalinist oppression due to their importance for the Soviet nuclear project, continued to flourish. Yuri Manin is one of key junior figures of the blossoming Moscow mathematical community, alongside Vladimir Arnold, Sergei Novikov, Yakov Sinai, Alexander Kirillov, Ernest Vinberg. His interests take him beyond mathematics: to the human sciences, the literary and cultural milieu. He is a close friend of Vladimir Vysotsky, the singer-songwriter and actor known to every single person in the USSR, and the brothers Arkady and Boris Strugatsky, the best-known science fiction writers of the country. The prestige of science is very high; a career in science offers an attractive safe haven in the tightly controlled Soviet society.

In 1960, Yuri Manin becomes a researcher at the Steklov Mathematical Institute of the Academy of Sciences of the USSR. He takes part in the study group organised by Shafarevich with the aim of giving a modern treatment to the results and methods of the Italian school of algebraic geometry. The outcome is the celebrated volume 'Algebraic surfaces' [10] published with the telling epigraph in ancient Greek taken from Athenaeus' 'Deipnosophists': "... Aeschylus declared that his tragedies were leftovers from Homer's great dinners". Manin's contribution to the project concerns rational and ruled surfaces; thus begins his long series of papers on the geometry, combinatorics, and arithmetic of geometrically rational surfaces summarised in his book 'Cubic forms' (1972) [7]. In his paper [3, Part I] published in 1966, he systematically studies geometrically rational surfaces, revisiting work of F. Enriques and B. Segre. Manin is interested in the existence of rational points and the validity of

[2] This 36-level skyscraper was the tallest building in Europe until 1990.

the Hasse principle, birational classification and birational invariants, the structure of the Cremona group and its twisted forms. He points out that for a smooth and projective surface X over a field k with separable closure $\overline{k}$, if we set $X_{\overline{k}} = X \times_k \overline{k}$ then the first Galois cohomology group $\mathrm{H}^1(k, \mathrm{Pic}(X_{\overline{k}}))$ with coefficients in the Picard group $\mathrm{Pic}(X_{\overline{k}})$ does not change under the blowup of a closed point, and thus is a birational invariant of X. At that time he does not seem to be aware of the close relation of $\mathrm{H}^1(k, \mathrm{Pic}(X_{\overline{k}}))$ to the Brauer group[3]. Manin observes that the group $\mathrm{H}^1(k, \mathrm{Pic}(X_{\overline{k}}))$ is easy to calculate; for a smooth cubic surface it can be computed from the partition of the set of 27 lines into orbits of the Galois group $\mathrm{Gal}(\overline{k}/k)$. To illustrate this technique, let $X_a \subset \mathbb{P}^3_k$ be the cubic surface

$$x_0^3 + x_1^3 + x_2^3 + a x_3^3 = 0,$$

where a is a nonzero element of k. If the field k contains cube roots of unity and a is not a cube in k, then $\mathrm{H}^1(k, \mathrm{Pic}(X_{a,\overline{k}})) \cong \mathbb{Z}/3 \times \mathbb{Z}/3$. Since $\mathrm{H}^1(k, \mathrm{Pic}(\mathbb{P}^2_{\overline{k}})) = 0$, we conclude that X_a is irrational over k. (A more general result, the irrationality of minimal cubic surfaces was obtained by B. Segre in 1951.) In his Tata Institute lectures (1966), Shafarevich asked for which $a, b \in k$ the surfaces X_a and X_b are birationally equivalent. In [3, Part II] Manin developes Segre's method and proves that two minimal cubic surfaces are birationally equivalent if and only if they are projectively isomorphic. Using this, an easy calculation shows that X_a and X_b are birationally equivalent over k if and only if a/b is a cube in k.

Manin reports on his work on geometrically rational surfaces in his talk 'Rational surfaces and Galois cohomology' at the International Congress of Mathematicians in Moscow (1966), see [4]. The footnote on p. 503 of [4] says that during the congress Michael Artin pointed out to him that the group $\mathrm{H}^1(k, \mathrm{Pic}(X_{\overline{k}}))$ associated to a geometrically rational surface X over a finite field k is isomorphic to the (cohomological) Brauer group $\mathrm{Br}(X)$ as defined by Grothendieck in 'Le groupe de Brauer II'; the isomorphism comes from the Leray spectral sequence for the étale cohomology attached to the morphism $X_{\overline{k}} \to X$. At the end of his talk Manin discusses work of François Châtelet and says: "...the Châtelet surfaces are of considerable interest. The technique now being used to study them strongly recalls the 'first descent' in the theory of elliptic curves; it will be very interesting to see whether this similarity is only a superficial one". This similarity has been confirmed in work of J.-L. Colliot-Thélène and J.-J. Sansuc who developed a general theory of descent on arbitrary varieties that generalises 'the first descent' on elliptic curves, using torsors for groups of multiplicative type.

In May and June of 1967, Yuri Manin participates in Alexandre Grothendieck's *Séminaire de géométrie algébrique* on intersection theory and the Riemann–Roch theorem (SGA 6) in IHES, Bures-sur-Yvette. A warm recollection of his discussions

[3] Grothendieck's Bourbaki talks 'Le groupe de Brauer I, II' were given in May and November 1965, whereas his longer text 'Le groupe de Brauer III' although dated March 1966 appeared together with the other two in 'Dix exposés sur la cohomologie des schémas' in 1968.

and correspondence with Grothendieck can be found in [8]. A result of this visit, Manin's paper [5] becomes the first ever publication on motives. Grothendieck liked it and recommended it to David Mumford as "a nice foundational paper".

Manin's visit to Paris had also other consequences. In [8] he writes: "When I brought xeroxed papers by Gino Fano back from Bures in 1967, Vassya Iskovskikh and I could read them without bothering much in which language they have been written, and then produce the first examples of birationally rigid varieties, and unirational but not rational threefolds using Fano methods". The joint paper with Vasily Iskovskikh [2] established that every birational map between smooth quartic threefolds in $\mathbb{P}^4_k$ over any field k is an isomorphism. It follows that the group of birational self-maps is finite. This property shows that no smooth quartic threefold can be rational. Examples of unirational quartic threefolds have been earlier constructed by Segre, so these provide examples of unirational but not rational varieties, giving a negative answer to Lüroth's problem. Fano claimed to have proved this result, but his arguments were incomplete. It seems that it is in this paper that smooth threefolds with ample anticanonical class were called 'Fano varieties' for the first time.

Meanwhile the political climate in Russia worsened. After the forced psychiatric confinement of the human rights activist and mathematician Alexander Esenin-Volpin, ninety-nine Soviet mathematicians signed a letter to the authorities asking for his release[4]. The list of signatories of this letter reads like 'Who's Who' of Russian mathematics. Esenin-Volpin was indeed released, but those who signed were punished by the authorities. In particular, the priviledge to travel abroad was revoked from those to whom it was previously accorded. However, a critical mass of mathematicians and abundance of strong students ensured the continuing flourishing of mathematics in Moscow, with seminars of Gelfand, Manin, Arnold (amongst others) serving as main focal points. When I became Manin's student in 1980, he was not allowed to do any undergraduate teaching, presumably because that would have brought him too close to the 'ideological frontline'. This may have been a blessing in disguise. Besides, he was not restricted in the choice of his 'special courses', seminars and study groups aimed at a smaller circle of his own students. The weekly Manin seminar was attended by about thirty people. No attempt was made at a diverse set of speakers. In the years when I attended it as a student, many of talks were given by Alexander Beilinson; nobody complained. But Manin's seminar was about much more than talks: it was a meeting place where mathematical ideas were passed around. In the breaks, participants walked in pairs in circular corridors of the Main Building of the university discussing mathematics. Many joint papers started in this way.

Perhaps the most characteristic feature of Manin's mathematical genius was his uncanny knack of seeing a far-reaching theoretical potential in mathematical facts and observations that could be perceived by others as specific and isolated, or merely

[4]The Russian text of the letter is available in https://math.ru/history/p99/index.htm

as part of a familiar picture. Manin generously shared his insights; his papers are scattered with open questions and suggestions. I can think of two major examples of his amazing insight in the areas with which I am more familiar: the Brauer–Manin obstruction (1970) and the Manin–Batyrev conjectures (1989-1991).

Prior to Manin's talk at the International Congress of Mathematicians in Nice[5] (1970) [6], several counterexamples to the Hasse principle were already known in the literature, usually based on various reciprocity laws or explicit calculation of class groups. Manin points out that the Brauer–Grothendieck group gives a completely general obstruction to the Hasse principle on a variety over a global field, thus allowing a uniform treatment of these various counterexamples. Fundamentally, the Brauer–Manin obstruction is a generalisation to arbitrary varieties of the Cassels–Tate pairing between the Tate–Shafarevich groups of an abelian variety and its dual. Manin introduces the now standard notation $\mathrm{Br}_1(X) \subset \mathrm{Br}(X)$ for the 'algebraic' part of the Brauer group, i.e. the kernel of the natural map $\mathrm{Br}(X) \to \mathrm{Br}(X_{\overline{k}})$, and $\mathrm{Br}_0(X)$ for the image of the Brauer group of the ground field $\mathrm{Br}(k)$ in $\mathrm{Br}(X)$. If k is a number field, the quotient $\mathrm{Br}_1(X)/\mathrm{Br}_0(X)$ can be identified with the already familiar group $\mathrm{H}^1(k, \mathrm{Pic}(X_{\overline{k}}))$. Manin discusses the problem of determining the image of the natural map $\mathrm{Br}(X) \to \mathrm{Br}(X_{\overline{k}})^G$, where $G = \mathrm{Gal}(\overline{k}/k)$. When k is a number field, he mentions the hypothetical finiteness of $\mathrm{Br}(X_{\overline{k}})^G$ in connection with the Tate conjecture. The Brauer–Manin obstruction revolutionised the theory of Diophantine equations by enabling one to study local-to-global principles for rational points beyond the narrow family of varieties that satisfy the Hasse principle, such as quadrics or principal homogeneous spaces of simply connected semisimple groups. Since Manin's foundational boost, through the efforts of Swinnerton-Dyer, Colliot-Thélène, Sansuc, and many, many others, the area of arithmetic geometry concerned with rational points has grown into a broad field combining geometric, cohomological and analytic methods. The Brauer group plays a prominent double role: it is a birational invariant, but also a group obstructing the local-to-global principle for rational points.

Another of Manin's lasting achievements in arithmetic geometry is his conjecture on the growth of the number of rational points of bounded height of varieties, known as the Manin conjecture, or Batyrev–Manin conjecture. For a long time, Manin felt a lack of analysis in the theory of rational points on surfaces. With the arrival of computers in the late 1980s, he started to experiment with counting rational points of bounded height on cubic surfaces. His favourite, still unsolved, problem was the following analogue of the Mordell–Weil theorem for cubic surfaces: is it possible to construct all rational points of a cubic surface from a finite list, using secants and tangents? The results in the literature obtained for higher-dimensional varieties by the circle method of Hardy and Littlewood (such as Birch's theorem) seemed to indicate that for a variety over $\mathbb{Q}$ embedded into a projective space by the

[5] which he was not allowed to attend, like those in Helsinki (1978) and Berkeley (1986) where he was invited

anticanonical linear system, the growth function is linear or close to linear. His first paper on this subject, written jointly with J. Franke and Yu. Tschinkel and published in *Inventiones* in 1989, suggests that the correct asymptotic is $cH(\log H)^t$, where $t = \operatorname{rk}\operatorname{Pic}(X) - 1$. Indeed, this is essentially compatible with taking products, and is proved in the paper for projective homogeneous spaces of semisimple groups. The follow-up paper with Victor Batyrev proposes a more precise form of the conjecture that takes into account the need to remove the so called ‘accumulating subvarieties’, and verifies this conjecture for some del Pezzo surfaces. The precise form of the constant c was worked out by Emmanuel Peyre. Although it soon became clear that the conjecture needs further refinement (one has to remove ‘thin’ subsets of rational points to avoid counterexamples), it attracted much interest of analytic number theorists and generated a staggering amount of research. In the long run, analytic results in the direction of this and other related conjectures reestablish the classical balance of algebraic and analytic methods in number theory.

For lack of space and expertise I leave it to others to discuss Manin’s papers on modular curves, cusp forms, p-adic L-functions and his ample work motivated by his keen interest in mathematical physics (instantons, Yang–Mills fields, supergeometry, quantum groups, quantum cohomology, operads, and much more).

In 1992–93 Manin spends a year in MIT. In 1993 he moves to the Max Planck Institute for Mathematics in Bonn to take up the position of a director. He is awarded a string of international prizes. He becomes director emeritus in 2005, without any slowing down in his research.

Yuri Ivanovich Manin died in Bonn on the 7 January 2023.

I would like to finish this subjective and highly incomplete sketch of Yuri Manin’s mathematical biography with a famous quote from T.S. Eliot’s ‘Four quartets’ that Manin used to described his attitude to mathematics and, perhaps, to life itself[6]:

“For us, there is only the trying. The rest is not our business”.

Acknowledgement The author is grateful to Jean-Louis Colliot-Thélène, Miles Reid and Yu.G. Zarhin for their comments on earlier drafts of this article.

References

[1] R. Coleman. Manin’s proof of the Mordell conjecture over function fields. *L’Enseign. Math.* (2) **36** (1990), no. 3-4, 393–427.

[2] V.A. Iskovskih and Yu.I. Manin. Three-dimensional quartics and counterexamples to the Lüroth problem. (Russian) *Mat. Sbornik* (N.S.) **86** (1971), 140–166.

[6] as communicated by Vladimir Drinfeld at the memorial event for Yu.I. Manin in the Max Planck Institute on the 13 January 2023

[3] Yu.I. Manin. Rational surfaces over perfect fields. (Russian) *Inst. Hautes Études Sci. Publ. Math.* **30** (1966), 55–113. Rational surfaces over perfect fields, II. (Russian) Mat. Sbornik (N.S.) **72** (1967) 161–192.

[4] Yu.I. Manin. Rational surfaces and Galois cohomology. *Proc. Internat. Congr. Math.* (Moscow, 1966), Mir, Moscow, 1968, 495–509.

[5] Yu.I. Manin. Correspondences, motifs and monoidal transformations. (Russian) *Mat. Sbornik* (N.S.) **77** (1968) 475–507.

[6] Yu.I. Manin. Le groupe de Brauer–Grothendieck en géométrie diophantienne. *Actes du Congrès International des Mathématiciens*, (Nice, 1970), Tome 1, pp. 401–411. Gauthier-Villars, Paris, 1971.

[7] Yu.I. Manin. *Cubic forms: algebra, geometry, arithmetic.* (Russian) Nauka, Moscow, 1972. English translation: *Cubic forms. Algebra, geometry, arithmetic.* Translated from the Russian by M. Hazewinkel. Second edition. North-Holland Mathematical Library, **4**. North-Holland, Amsterdam, 1986.

[8] Yu.I. Manin. Forgotten motives: the varieties of scientific experience. *ICCM Notices* **2** (2014), no. 1, 6–10.

[9] Z. Miller. *From experience. A story of an evacuation.* (Russian) https://berkovich-zametki.com/2014/Starina/Nomer1/Manin1.php

[10] I.R. Shafarevich, B.G. Averbuh, Yu.R. Vainberg, A.B. Zhizhchenko, Yu.I. Manin, B.G. Moishezon, G.N. Tiurina, A.N. Tiurin. Algebraic surfaces. (Russian) *Trudy Mat. Inst. Steklov* **75** (1965) pp. 1–215.