

Algebra III M3P8, M4P8

Test 2

5 December 2017

You are asked to justify your answers. (You can use all results from lectures.)

1. For each of the prime numbers $p = 2, 3, 5, 7$ determine whether the principal ideal generated by $x^2 + 1$ is a maximal ideal of the polynomial ring $\mathbb{F}_p[x]$, where \mathbb{F}_p is a finite field with p elements.
2. Let α be the real root of the equation $x^7 - 5 = 0$. Determine $[\mathbb{Q}(\alpha) : \mathbb{Q}]$, with a complete proof.
3. Determine $[\mathbb{Q}(e^{\frac{\pi i}{4}}) : \mathbb{Q}]$, with a complete proof.