Algebra III M3P8, M4P8

Test 1

November 14, 2017

You are asked to justify your answers. (A short explanation will suffice.)

1. Using Euclid's algorithm, or otherwise, find a greatest common divisor of 2 and $3-i$ in the ring $\mathbb{Z}[i]$.

2. For each of the following elements of $\mathbb{Z}[\sqrt{2}]$ determine if the element is a unit, an irreducible, or neither:

$$\sqrt{2}, \quad 1+\sqrt{2}, \quad 2+\sqrt{2}, \quad 3+\sqrt{2}, \quad 4+\sqrt{2}.$$

3. Let $I \subset \mathbb{Z}[x]$ be the set of polynomials $f(x)=\sum_{i=0}^{n} a_i x^i$ such that $a_i \in \mathbb{Z}$ and a_0, a_1, a_2 are multiples of 3.
(a) Show that I is an ideal in $\mathbb{Z}[x]$.
(b) Find $f_1(x), f_2(x) \in \mathbb{Z}[x]$ such that $I = f_1(x)\mathbb{Z}[x] + f_2(x)\mathbb{Z}[x]$.
(c) Does there exist $g(x) \in \mathbb{Z}[x]$ such that $I = g(x)\mathbb{Z}[x]$?