Algebra III M3P8, M4P8

Test 2

1.

(a) No: the images of x and $x-1$ in R/I are zero divisors.
(b) No: the images of x and y in R/I are zero divisors.
(c) Yes: we have $\mathbb{Q}[x,y]/(x-y) \cong \mathbb{Q}[x]$ which is an integral domain.
(b) Yes: $\mathbb{Q}[x,y]/(2x-1, x^2+y^2-1) \cong \mathbb{Q}[y]/(y^2-3/4) \cong \mathbb{Q}(\sqrt{3})$, which is a field.

2.

α is not in $\mathbb{Z}/2$, so the minimal polynomial of α cannot be linear. If this polynomial is quadratic, then F contains the subfield $\mathbb{Z}/2(\alpha)$ of degree 2 over $\mathbb{Z}/2$ which is impossible since F has degree 3 over $\mathbb{Z}/2$. Now α is a root of x^3+x+1, so this is it.