

# Algebra III M3P8, M4P8

## Test 2

1. Which of these are prime ideals?

(a)  $R = \mathbb{Q}[x]$ ,  $I = \{f(x) \in R \mid f(0) = f(1) = 0\}$ .

(b)  $R = \mathbb{Q}[x, y]$ ,  $I = (xy)$ .

(c)  $R = \mathbb{Q}[x, y]$ ,  $I = (x - y)$ .

(b)  $R = \mathbb{Q}[x, y]$ ,  $I = (2x - 1, x^2 + y^2 - 1)$ .

2. The polynomial  $x^3 + x + 1$  is irreducible over  $\mathbb{Z}/2$ , so let  $F$  be the field  $\mathbb{Z}/2[x]/(x^3 + x + 1)$ . Let  $\alpha \in F$  be the coset  $x^2 + (x^3 + x + 1)$ . Find the minimal polynomial of  $\alpha$  over  $\mathbb{Z}/2$ .