Algebra III M3P8, M4P8

Test 1 Solutions

Full marks will not be given, unless your proof is complete.

1. **7 marks**

Divide $7+6i$ by $5+5i$. The result is $\frac{13}{10} - i\frac{1}{10}$. The closest element of $\mathbb{Z}[i]$ is 1, so we write $7+6i = 1(5+5i) + (2+i)$. Now divide $5+5i$ by $2+i$. The result is $3+i \in \mathbb{Z}[i]$, so that $2+i$ divides $5+5i$. The last non-zero remainder is $2+i$, so this is a greatest common divisor of $5+5i$ and $7+6i$. Remember that it is not unique so that $\pm(2+i)$ and $\pm(-1+2i)$ are all greatest common divisors of $5+5i$ and $7+6i$.

2. **7 marks**

By the solution of Question 1 we have $7+6i = (2+i)(4+i)$. The norm of $2+i$ is 5, so it is not a unit. 5 is prime, so that if $2+i$ is a product of two elements of $\mathbb{Z}[i]$, then the norm of one of the factors is 1, and this factor must be a unit. Therefore, $2+i$ is irreducible. The norm of $4+i$ is also prime, so a similar argument shows that $4+i$ is irreducible.

3. **6 marks**

The condition $(a+bx)(c+dx) = 1$ is equivalent to $ac = 1$, $ad+bc = 0$, $bd = 0$. Multiplying the second equality by $a = c^{-1}$ we get $a^2d + b = 0$. Multiplying this by b we get $b^2 = 0$ since $bd = 0$.