

# Algebra III M3P8, M4P8

## Test 1

1. Using Euclid's algorithm, or otherwise, find a greatest common divisor of  $5 + 5i$  and  $7 + 6i$  in the ring  $\mathbb{Z}[i]$ . (You must show your working.)
2. Using the calculations of Question 1, or otherwise, write  $7 + 6i$  as a product of irreducibles in the ring  $\mathbb{Z}[i]$ . (You need to justify that the factors are irreducibles.)
3. Let  $R$  be a commutative ring with 1 (but not necessarily an integral domain). Let  $a, b, c, d \in R$  be such that the product of  $a + bx$  and  $c + dx$  in the polynomial ring  $R[x]$  is the constant polynomial 1. Prove that  $b$  is a nilpotent in  $R$ .