Question 2.

(a) Sketch the curve $y = (2x+1)\exp(-x^2)$. Label any turning points explicitly. Use Rolle's theorem to identify an x-interval in which there must be a point of inflection. (You need not find it exactly).

(b) Sketch the curve given in polar coordinates (r, θ) by

$$r = \sec(\theta - \alpha) \qquad \text{where } \alpha \text{ is a given constant with } 0 < \alpha < \pi/2.$$

(a) As $x \to \pm\infty$, $y \to 0$ quickly. For $x > -1/2$, $y > 0$, while $y < 0$ for $x < -1/2$. Differentiating,

$$y' = \exp(-x^2)\,[-2x(2x+1)+2] = 2\exp(-x^2)[1-x-2x^2] = \exp(-x^2)(1-2x)(1+x)$$

which is zero at $x = 1/2$ and $x = -1$. As x increases through $x = -1$, the sign of y' changes from $-$ to $+$ so this is a minimum. Similarly at $x = 1/2$, the sign changes from $+$ to $-$ giving a maximum. So there is a maximum at $(1/2, 2e^{-1/4})$ and a minimum at $(-1, -1/e)$.

From the shape of the graph there must be a point of inflection ($y'' = 0$) between the two turning points. This follows from Rolle's theorem. As $y' = 0$ at two points, y'' must be zero somewhere between these two points, i.e. somewhere in $(-1, 1/2)$. (Obviously y' is continuous and differentiable.) **[5 marks]**

(b) If we rotate the x-axis through α, the equation becomes $1 = r\cos\theta = x$. So the curve must be the straight line perpendicular to the line $\theta = \alpha$ a unit distance from the origin. Alternatively,

$$1 = r\cos(\theta - \alpha) = r\cos\theta\cos\alpha + r\sin\theta\sin\alpha = x\cos\alpha + y\sin\alpha.$$

This is the straight line joining the points $(\sec\alpha, 0)$ and $(0, \operatorname{cosec}\alpha)$. **[5 marks]**