

(a) Neither: $f(x)$ is not even defined for $x < 0$, so f cannot be even or odd. [1]

(b) The largest possible domain of f is $[0, \infty)$. The function is defined for all non-negative x , i.e. $x \geq 0$. Note the denominator is never zero. [1]

As x increases from zero, the denominator increases from 1 to infinity. So the value of f decreases from 1 to 0. Thus the range of f is $(0, 1]$ or $0 < x \leq 1$. [2]

(c) To find the inverse, we set

$$y = \frac{1}{1 + \sqrt{x}} \quad \implies \quad \sqrt{x} = \frac{1}{y} - 1.$$

Squaring, we have

$$x = f^{-1}(y) \equiv \left(\frac{1}{y} - 1\right)^2 = \frac{(1-y)^2}{y^2}. \quad [4]$$

(d) Now

$$f(f^{-1}(x)) = \frac{1}{1 + \sqrt{f^{-1}(x)}} = \frac{1}{1 + \left|\frac{1}{x} - 1\right|} = \frac{1}{1 + 1/x - 1} = x, \quad [2]$$

since $0 < x \leq 1$ (the **domain** of f^{-1} is the same as the **range** of f).

[Note $\sqrt{t^2} \neq t$, as t may be negative. Strictly, $\sqrt{t^2} \equiv |t|$.]

Total: [10]

Note to markers: They have ten minutes allocated for this question. I predict many will blur the distinction between, for example, $x < 1$ and $x \leq 1$ in part (b). And many may forget to check that $(1/x - 1)$ is positive before taking the “convenient” square root in part (d). It’s up to you to decide whether to deduct a mark for these and similar errors. Do correct inaccuracies on the scripts even if you decide not to penalise them. This sheet will be made available to the students in due course. JM